(7 Pages) **Reg. No. :** ....

Code No. : 6318 **Sub. Code : PMAM 33**

M.Sc. (CBCS) DEGREE EXAMINATION,
NOVEMBER 2021

Third Semester

Mathematics — Core

ADVANCED ALGEBRA — I

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answers :

1. If V is a vector space than its dual space is

(a) Hom (V,V) (b) Hom (F,V)

(c) Hom (V,F) (d) Hom (F,F)

2. An orthonormal set consists of

(a) zero vector

(b) unit vector

(c) linearly dependent vector

(d) inner products

3. If $S, T \in A(V)$ and S is regular then $r(ST) =$

(a) $r(S)$ (b) $r(T)$

(c) 1 (d) 0

4. If $\lambda - 1$ is singular then λ is

(a) also singular (b) regular

(c) an eigen-value (d) zero

5. The invariants of $\begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ are

(a) 1, 1 (b) 5, 4

(c) 2, 1 (d) 3, 2

6. If W is a subspace of V and $T \in A(V)$ are $WT \subset W$, then W

(a) equal T

(b) variant under T

(c) invariant under T

(d) has no other subspaces

7. Trace of A is defined when A is a ———— matrix.

(a) triangular (b) symmetric

(c) square (d) skew-symmetric

8. If the matrix B is obtained from A by a permutation, which is odd, of the rows of A then $\det A =$

(a) $\det B$ (b) $-\det B$

(c) 0 (d) 1

9. If T is $A(V)$ is Hermitian then all its characteristic roots are

(a) real (b) imaginary

(c) 0 (d) 1

10. If all the characteristic roots of a normal transformation are of absolute value 1, then it is

(a) identity (b) symmetric

(c) transitive (d) unitary

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b)

11. (a) Show that if $\dim V = m$ then $\dim Hom(V,V) = m^2$.

Or

(b) State and prove the Schwartz inequality on inner product spaces.

12. (a) If $S,T \in A(V)$ and if S is regular, prove that T and STS^{-1} have the same minimal polynomial.

Or

(b) If V is finite dimensional over F, then prove that $T \in A(V)$ is regular if and only if T maps V onto V.

13. (a) If M, of dimension m, is cyclic with respect to T, then prove that $\dim MT^k$ is $m-k$.

Or

(b) Suppose $V = V_1 \oplus V_2$, where V_1, V_2 are subspaces of V invariant under T. If $T_1 T_2$ are linear transformation induced by T on V_1 and V_2, with minimal polynomials $p_1(x)$ and $p_2(x)$, respectively, show that the minimal polynomial of T is the lcm of $p_1(x)$ and $p_2(x)$.

14. (a) Prove that if all the elements in one row of A in F_n are multiplied by τ in F, then $\det A$ is multiplied by τ.

Or

(b) If two elements of A are equal, show that $\det A = 0$, where A is an m × n matrix.

15. (a) Prove that the linear transformation T on V is unitary if and only if it takes an orthonormal basis of V onto an orthonormal basis of V.

Or

(b) If T is Hermitian and $vT^k = 0$ for all $k \geq 1$ then prove that $vT = 0$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) If there is a homogeneous system of m equations in n unknowns with $n > m$, prove that if has a non-trivial solution.

Or

(b) If W is a subspace of a finite dimensional vector space V, then prove that V is the direct sum of W and its orthogonal complement.

17. (a) If $\lambda_1, \lambda_2, \ldots, \lambda_k$ in F are distinct characteristic roots of T in $A(V)$ and is $v_1, v_2, \ldots, v_k$ are characteristic vectors belonging to $\lambda_1, \lambda_2, \ldots, \lambda_k$ respectively, show that $v_1, v_2, \ldots, v_k$ are linearly independent.

Or

(b) Show that $A(V)$ and F_n are isomorphic algebras.

18. (a) If $T \in A(V)$ has all its characteristics roots in F, show that this is a basis of V in which the matrix of T is triangular.

Or

(b) If $T \in A(V)$ is nilpotent, prove that there exists a subspace of W of V, invariant under T, such that $V = V_1 \oplus W, V_1$ is spanned by $v, vT, \ldots\ldots, vT^{n_1 - 1}$.

19. (a) If F is a field of characteristic 0, and if $trT^i = 0$ for all $i \geq 1$, prove that T is nilpotent.

Or

(b) For A, B in F_n, prove that $\det(AB) = \det(A)\det(B)$.

20. (a) If $\{v_1, v_2, \ldots, v_n\}$ is an orthonormal basis of V and if (α_{ij}) is the matrix of T in $A(V)$, prove that the matrix of T^* in this basis is (β_{ij}) where $\beta_{ij} = \overline{\alpha_{ij}}$.

Or

(b) If N is a normal linear transformation on V, prove that there exists an orthonormal basis in which the matrix of N is diagonal.

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