(6 pages)

Reg. No. : ....

Code No. : 20579 E **Sub. Code : SMMA 62**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2021.

Sixth Semester

Mathematics — Core

NUMBER THEORY

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer.

1. The value of $1+2+3+....+n$ is __________.

   (a) $\frac{n(n+1)}{2}$ (b) $\frac{n(n+1)}{3}$

   (c) $n(n+1)$ (d) $\frac{n(n+1)}{6}$

2. In the Pascal's triangle, the $(k+1)^{th}$ number in the n^{th} row is

   (a) $\binom{n}{k}$ (b) $\binom{n}{k+1}$

   (c) $\binom{n}{k-1}$ (d) $\binom{n}{k+2}$

3. The gcd (119, 272) is

(a) 27 (b) 9

(c) 17 (d) 57

4. For any integer $k \neq 0$, gcd (ka, kb) = ?

(a) k gcd (a, b) (b) $|k|$ gcd (a, b)

(c) gcd (a, b) (d) k^2 gcd (a, b)

5. According to division algorithm, every positive even integer can be uniquely written as

(a) $4n+1$ (b) $4n+3$

(c) $4n$ or $4n+2$ (d) none of these

6. Which of the following is irrational?

(a) $3^{1/2}$ (b) $11^{1/2}$

(c) $4^{1/4}$ (d) All the above

7. If 5^{48} is divided by 12, then the remainder is

(a) 1 (b) 2

(c) 4 (d) 9

8. A solution of the linear congruence $5x \equiv 2 \pmod{26}$ is

(a) 10 (b) 12

(c) 14 (d) 16

9. The least odd prime for which the congruence $(p-1)! \equiv -1 \pmod{p^2}$ holds good is

(a) 5 (b) 7

(c) 11 (d) 13

10. The number of pseudo primes is

(a) 0

(b) 1

(c) more than 1 but finite

(d) infinite

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that $1^2 + 3^2 + 5^2 + \ldots + (2n-1)^2 = \dfrac{4n^3 - n}{3}$ $\forall\ n \geq 1$.

Or

(b) Define a triangular number. Give an example prove that the sum of the first n natural numbers is triangular.

12. (a) State and prove the Euclid's Lemma.

Or

(b) If $\gcd(a, b) = 1$ prove that $\gcd(a+b, a^2 - ab + b^2) =$ 1 or 3.

13. (a) If $n > 1$, show that $n^2 + 4$ is composite.

Or

(b) Verify that the integers 1949 and 1951 are twin primes.

14. (a) Prove that for arbitrary integers a and b, $a \equiv b \pmod{n}$ if and only if a and b leave the same non-negative remainder when divided by n.

Or

(b) Find the last two digits of the number. 9^{9^9}

15. (a) State and prove Fermat's Theorem.

Or

(b) If P is a prime, prove that for any integer a, $P \mid a^p + (p-1)!$ and $P \mid (p-1)! a^p + a$.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Establish the binomial theorem.

Or

(b) (i) State and prove Archimedean Property.

(ii) State and prove the first principle of finite induction.

17. (a) State and prove the Division Algorithm.

Or

(b) Solve the linear Diophantine equation $180x + 75y = 9000$.

18. (a) (i) State and prove the fundamental theorem of Arithmetic.

(ii) Prove that $48 \mid m(m^2 + 20)$.

Or

(b) Discuss about the Goldbach conjecture.

19. (a) (i) If, $ca \equiv cb(\bmod n)$ prove that $a \equiv b\left(\bmod \frac{n}{d}\right)$ when d = G.C.D. (c.n.).

(ii) What is the remainder when the sum $1! + 2! + 3! + \ldots + 99! + 100!$ is divided by 12.

Or

(b) If $(a, m) \mid b$ prove that $ax \equiv b(\bmod m)$ has exactly (a, m) solutions.

20. (a) If P is a prime, then $a^P \equiv a(\bmod P)$ for any integer a.

Or

(b) If n is an odd pseudo prime, then prove that $Mn = 2^n - 1$ is larger one.
