(8 pages) **Reg. No. :** ....

Code No. : 41188 E Sub. Code : JMMA 6 A

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2019.

Sixth Semester

Mathematics — Main

Major Elective III– Optional – FUZZY MATHEMATICS

(For those who joined in July 2016 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. A set whose members can be labelled by the positive integers is called a
   (a) Open set (b) Closed set
   (c) Countable set (d) Uncountable set.

2. The scalar cardinality of the fuzzy set $A = \cdot 4/v + \cdot 2/w + \cdot 5/x + \cdot 4/y + 1/z$ is
   (a) 1.5 (b) 2.5
   (c) 2.1 (d) 1.9

3. The α – cut of the complement of A is always the same as the complement of the
   (a) Strong α-cut of A
   (b) α–cut of A
   (c) Strong $(1-\alpha)$-cut of A
   (d) $(1-\alpha)$-cut of A.

4. Let $f : X \to Y$ be an arbitrary crisp function. Then for any $A_i \in \mathfrak{I}(X)$, $i \in I$.
   (a) $f\left(\bigcap_{i \in I} A_i\right) \subseteq \bigcap_{i \in I} f(A_i)$
   (b) $f\left(\bigcap_{i \in I} A_i\right) \supseteq \bigcap_{i \in I} f(A_i)$
   (c) $f\left(\bigcap_{i \in I} A_i\right) = \bigcap_{i \in I} f(A_i)$
   (d) $f\left(\bigcap_{i \in I} A_i\right) \subset \bigcap_{i \in I} f(A_i)$

5. The standard fuzzy intersection is the only ———————— t-norm.
   (a) Archimedean
   (b) Strictly Archimedean
   (c) Idempotent
   (d) Involutive

6. If $W = \langle \cdot 3, \cdot 1, \cdot 2, \cdot 4 \rangle$, then $h_W(\cdot 6, \cdot 9, \cdot 2, \cdot 7) =$
   (a) .54 (b) 5·4
   (c) ·45 (d) 4·5.

7. If $A = [0, 1]$, $B = [1, 2]$ and $C = [-2, -1]$, then $A \cdot (B + C) =$
   (a) $[-1, 1]$ (b) $[-2, 2]$
   (c) $[1, 1]$ (d) $[2, 2]$.

8. In (v, T, X, g, m), g is a

(a) syntactic rule

(b) semantic rule

(c) linguistic term

(d) name of the variable.

9. In a linear programming problem the function to be minimized or maximized is called

(a) Constraint function

(b) Goal function

(c) An objective function

(d) Feasible function.

10. The set of vectors X that satisfy all given constraints is called a

(a) cost vector

(b) feasible set

(c) constraint matrix

(d) right-hand-side vector.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If $X = [0, 80]$ and $A : X \to [0, 1]$ defined by

$$A(x) = \begin{cases} 1 & \text{when } x \le 20 \\ \dfrac{(35 - x)}{15}, & \text{when } 20 < x < 25 \\ 0 & \text{when } x \ge 35 \end{cases}$$

then find

(i) ${}^{\circ}A$

(ii) ${}^{\alpha}A$

(iii) ${}^{\alpha+}A$

(iv) ${}^{1+}A$.

Or

(b) Prove that : A fuzzy set A on $\boldsymbol{R}$ is convex iff $A(\lambda x_1 + (1 - \lambda) x_2) \ge \min [A(x_1), A(x_2)]$ for all $x_1, x_2 \in \boldsymbol{R}$ and all $\lambda \in [0, 1]$, where min denotes the minimum operator.

12. (a) Let $A, B \in \mathcal{F}(X)$. Then for all $\alpha \in [0, 1]$, prove that

(i) $A \subseteq B$ iff ${}^{\alpha}A \subseteq {}^{\alpha}B$

(ii) $A \subseteq B$ iff ${}^{\alpha+}A \subseteq {}^{\alpha+}B$.

Or

(b) Let $f : X \to Y$ be an arbitrary crisp function. Then, prove that for any $A \in \mathcal{F}(X)$ and all $\alpha \in [0, 1]$ the following properties of f fuzzified by the extension principle hold :

(i) ${}^{\alpha+}[f(A)] = f({}^{\alpha+}A)$;

(ii) ${}^{\alpha+}[f(A)] \supseteq f({}^{\alpha}A)$.

[P.T.O.]

13. (a) Prove that every fuzzy complement has atmost one equilibrium.

Or

(b) For all $a, b \in [0, 1]$, prove that $i_{\min}(a, b) \leq i(a, b) \leq \min(a, b)$.

14. (a) If $A(x) = \begin{cases} 0 & \text{for } x \leq -1 \text{ and } x > 3 \\ (x+1)/2 & \text{for } -1 < x \leq 1 \\ (3-x)/2 & \text{for } 1 < x \leq 3 \end{cases}$

$B(x) = \begin{cases} 0 & \text{for } x \leq -1 \text{ and } x > 5 \\ (x-1)/2 & \text{for } 1 < x \leq 3 \\ (5-x)/2 & \text{for } 3 < x \leq 5 \end{cases}$

then find

(i) ${}^{\alpha}(A+B)$ (ii) ${}^{\alpha}(A-B)$

(iii) ${}^{\alpha}(A \cdot B)$ (iv) ${}^{\alpha}(A/B)$.

Or

(b) Let MIN and MAX be binary operations on R defined by

$$MIN(A, B)(z) = \sup_{z = \min(x, y)} [A(x), B(y)]$$

and

$$MAX(A, B)(z) = \sup_{z = \max(x, y)} [A(x), B(y)]$$

respectively. Then prove that, for any $A, B, C \in R$,

$$MIN[A, MAX(B, C)] = MAX[MIN(A, B), MIN(A, C)]$$

15. (a) Solve the following fuzzy linear programming problem :

Max. $Z = 5x_1 + 4x_2$

s.t. $\langle 4, 2, 1\rangle x_1 + \langle 5, 3, 1\rangle x_2 \leq \langle 24, 5, 8\rangle$

$\langle 4, 1, 2\rangle x_1 + \langle 1, .5, 1\rangle x_2 \leq \langle 12, 6, 3\rangle$

$x_1, x_2 \geq 0$.

Or

(b) Define Fuzzy linear programming problem.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Define the following :

(i) α-cut

(ii) strong α-cut

(iii) level set

(iv) support of a fuzzy set A

(v) The height of A

(vi) Normal and subnormal fuzzy sets.

Or

(b) Write down all the fundamental properties of crisp set operations.

17. (a) State and prove first decomposition theorem.

Or

(b) Let $A, B \in \mathcal{F}(X)$. Then prove that for all $\alpha, \beta \in [0,1]$.

(i) ${}^{\alpha}(A \cap B) = {}^{\alpha}A \cap {}^{\alpha}B$ and

${}^{\alpha}(A \cup B) = {}^{\alpha}A \cup {}^{\alpha}B$

(ii) ${}^{\alpha+}(A \cap B) = {}^{\alpha+}A \cap {}^{\alpha+}B$ and

${}^{\alpha+}(A \cup B) = {}^{\alpha+}A \cup {}^{\alpha+}B$.

18. (a) State and prove second characterization theorem of fuzzy complements.

Or

(b) Given a t-norm i and an involutive fuzzy complement C, the binary operation u on $[0,1]$ defined by $u(a,b) = c(i\,(C(a), C(b)))$ for all $a, b \in [0,1]$ is a t-conorm such that $< i, u, c >$ is a dual triple.

19. (a) Calculate the following :

(i) $[-1, 2] + [1, 3]$

(ii) $[-2, 4] - [3, 6]$

(iii) $[-3, 4]\,[-3, 4]$

(iv) $[-4, 6] / [1, 2]$.

Or

(b) Let $* \in \{+, -, \bullet, /\}$, and let A, B denote continuous fuzzy numbers. Then, prove that, the fuzzy set $A * B$ defined by $(A * B)(z) = \sup_{z = x * y} \min[A(x), B(y)]$ is a continuous fuzzy numbers.

20. (a) Explain multiperson decision making.

Or

(b) Solve the following fuzzy linear programming problem.

Max $Z = .5x_1 + .2x_2$

S.T.

$x_1 + x_2 \le B_1$

$2x_1 + x_2 \le B_2$

$x_1, x_2 \ge 0$.

where

$$B_1(x) = \begin{cases} 1 & for\ x \le 300 \\ \dfrac{400 - x}{100} & \text{for } 300 < x \le 400 \\ 0 & \text{for } x > 400 \end{cases}$$

and

$$B_2(x) = \begin{cases} 1, & \text{for } x \le 400 \\ \dfrac{500 - x}{100} & \text{for } 400 < x \le 500 \\ 0 & \text{for } x > 500 \end{cases}$$
