(8 pages) **Reg. No. :** ....................................

**Code No. : 6437** **Sub. Code : ZCSM 13**

M.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2022.

First Semester

Computer Science – Core

MATHEMATICAL FOUNDATION FOR COMPUTER SCIENCE

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer :

1. The ———— of two statements $P$ and $Q$ is the statement $P \wedge Q$.

   (a) disjunction

   (b) normal

   (c) conjunction

   (d) negation

2. A formula which is equivalent to a given formula and which consists of a sum of elementary products is called a ———— of the given formula.

   (a) disjunctive normal form

   (b) conjuctive normal form

   (c) predicate logic

   (d) inference

3. Let $R$ be a relation in a set $X$. If for every $x$ in $X(x, x) \in R$ then $R$ is called

   (a) reflexive (b) symmetric

   (c) transitive (d) antisymmetric

4. The power set of $\{a\}$ is

   (a) $\phi$ (b) $\{\phi, \{a\}\}$

   (c) $\{\phi\}$ (d) $\{a\}$

5. The rank of every $n$-square non-singular matrix is

   (a) $n$ (b) $n-1$

   (c) 1 (d) 0

6. Which of the following is called the characteristic equation of a matrix $A$?

(a) $|A-\lambda I|$ (b) $|A|=0$

(c) $A=0$ (d) $|A-\lambda I|=0$

7. A graph $G$ is said to be ———— if there is atleast one path between every pair of vertices in $G$.

(a) disconnected (b) component

(c) connected (d) hamiltonian

8. A ———— is defined as a finite alternating sequence of vertices and edges, beginning and ending with vertices such that each edge is incident with the vertices preceding and following it.

(a) path (b) circuit

(c) walk (d) subgraph

9. If $n$, $e$ and $k$ are the number of vertices, number of edges and number of components of a graph $G$, then its nullity =

(a) $n-k$ (b) $e-n$

(c) $e+k$ (d) $e-n+k$



10. In the adjacency matrix, if there is an edge between $i^{th}$ and $i^{th}$ vertices, then $x_{ij}$ =

(a) 0 (b) −1

(c) 2 (d) 1

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Obtain the principal disjunctive normal form of $\neg P \vee Q$.

Or

(b) Show that

$$(x)(P(x)\to Q(x))\wedge(x)(Q(x)\to R(x))\Rightarrow(x)(P(x)\to R(x))$$

12. (a) For any three sets $A, B$ and $C$, prove that $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$.

Or

(b) Let $R=\{(1,2),(3,4),(2,2)\}$ and $S=\{(4,2),(2,5),(3,1),(1,3)\}$. Find $R\circ S$, $S\circ R$, $(R\circ S)\circ R$, $R\circ(S\circ R)$ and $R\circ R$.

13. (a) Find the rank of the matrix $\begin{bmatrix} 6 & 1 & 3 & 8 \\ 4 & 2 & 6 & -1 \\ 10 & 3 & 9 & 7 \\ 16 & 4 & 12 & 15 \end{bmatrix}$.

Or

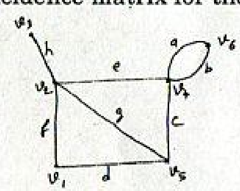
(b) Find the characteristics equation of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & 1 \end{bmatrix}$.

14. (a) Prove that in a connected graph $G$ with exactly 2k odd vertices, there exist $k$ edge-disjoint subgraphs such that they together contain all edges of $G$ and that each is a unicursal graph.

Or

(b) Prove that a connected graph $G$ is an Euler graph if and only if it can be composed into circuits.

15. (a) Find incidence matrix for the following graph.




Or

(b) Prove that in any tree with two or more vertices, there are atleast two pendant vertices.



PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Obtain conjunctive normal forms :

(i) $P \wedge (P \to Q)$

(ii) $\urcorner(P \vee Q) \rightleftarrows (P \wedge Q)$

Or

(b) Show that from

(i) $(\exists x)(F(x) \wedge S(x)) \to (y)(M(y)) \to W(y)$

(ii) $(\exists y)(M(y) \wedge \urcorner W(y))$.

the conclusion $(x)(F(x) \to \urcorner s(x))$ follows.

17. (a) (i) If $A = \{\alpha, \beta\}$ and $B = \{1, 2, 3\}$ , what are $A \times B$, $B \times A$, $A \times A$, $B \times B$ and $(A \times B) \cap (B \times A)$?

(ii) For any two sets $A$ and $B$, prove that $A \cup B = (A \cap \sim B) \cup (B \cap \sim A) \cup (A \cap B)$.

Or

(b) (i) Let $X = \{1, 2, 3\}$ and $f, g, h$ and $s$ be functions from $X$ to $X$ given by

$f = \{(1, 2), (2, 3), (3, 1)\}$

$g = \{(1, 2), (2, 1), (3, 3)\}$

$h = \{(1, 1), (2, 2), (3, 1)\}$

$s = \{(1, 1), (2, 2), (3, 3)\}$

find $f \circ g$, $g \circ f$, $f \circ h \circ g$ and $s \circ g$.

(ii) Let $X = \{1, 2, \ldots 7\}$ and $R = \{(x, y)/x - y \text{ is divisible by } 3\}$. Show that $R$ is an equivalence relation.

18. (a) Solve the system of equations

$$x + 2y + 3z = 10$$
$$2x - 3y + z = 1$$
$$3x + y - 2z = 9$$

Or

(b) Find the eigen values and eigen vectors of $\begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$.



19. (a) Prove that a simple graph with $n$ vertices and $k$ components can have atmost $(n-k)(n-k+1)/2$ edges.

Or

(b) (i) Define a complete graph.

(ii) Prove that in a complete graph with $n$ vertices there are $\frac{n-1}{2}$ edge-disjoint Hamiltonian circuits if $n$ is an odd number $\geq 3$.

20. (a) (i) Define rank of a graph.

(ii) Prove that every tree has either one or two centres.

Or

(b) (i) Define a circuit matrix.

(ii) Let $B$ and $A$ be, respectively, the circuit matrix and the incidence matrix of a self-loop-free graph whose columns are arranged using the same order of edges. Then prove that every row of $B$ is orthogonal to every $A$.

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