(6 pages) **Reg. No. :**..

Code No. : 30335 E **Sub. Code : JMMA 31/ JMMC 31/SMMA 31**

B.Sc.(CBCS) DEGREE EXAMINATION, NOVEMBER 2020.

Third Semester

Mathematics/Mathematics with CA — Main

REAL ANALYSIS — I

(For those who joined in July 2016 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. $(a, +\infty) =$ ——————

   (a) $\{x : x > a\}$ (b) $\{x : x \geq a\}$

   (c) $\{x : x < a\}$ (d) $\{x : x \leq a\}$

2. Every non-empty set of real numbers which is bounded above has a ——————

   (a) infimum (b) supremum

   (c) prime number (d) rational number

3. The sequence $\left((-1)^n\right)$ is a

(a) bounded sequence

(b) convergent sequence

(c) monotonic sequence

(d) divergent sequence

4. The limit of the sequence $\left(\frac{1}{n}\right)$ is

(a) 1 (b) n

(c) 0 (d) 2

5. A monotonic increasing sequence which is bounded above converges to its

(a) l.u.g. (b) g.l.b

(c) $-\infty$ (d) ∞

6. Which is Cauchy sequence?

(a) $\left((-1)^n\right)$ (b) (n)

(c) (n^n) (d) $(1/n)$

7. The series $1 + r + r^2 + \cdots + r^n + \cdots$ converges if

(a) $r > 1$ (b) $0 \le r < 1$

(c) $r = 1$ (d) $r < -1$

8. The series $\sum \frac{1}{n^p}$ diverges if

(a) $p > 1$ (b) $p < 1$

(c) $p = 1$ (d) $p \leq 1$

9. If Σa_n converges, then $\sum \frac{a_n}{n}$

(a) convergent (b) divergent

(c) oscillatory (d) none

10. Radius of convergence of binomial series is

(a) 0 (b) 1

(c) n (d) ∞

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 250 words.

11. (a) $|x| \leq a$ $(a \geq 0)$ if and only if $-a \leq x \leq a$.

Or

(b) State and prove Cauchy – Schwarz inequality.

12. (a) If $(a_n) \to a, (b_n) \to b$, then show that $(a_n b_n) \to ab$.

Or

(b) Prove : $(a_n) \to \infty, a_n \neq 0 \ \ \forall n \in N \Rightarrow \left(\frac{1}{a_n}\right) \to 0$ and show that the converse of this theorem is not true.

13. (a) If $a_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$, show that the sequence (a_n) diverges to ∞.

Or

(b) Prove that

$$\lim_{n \to \infty} \left\{ \frac{1}{n} [(n+1)\ (n+2) \cdots (n+1)]^{1/n} \right\} = 4/e .$$

14. (a) Discuss the convergence of the series $\sum \frac{n^3 + a}{2^n + a}$.

Or

(b) Discuss the convergence of the series $\sum \frac{1^2 + 2^2 + \cdots + n^2}{n^4 + 1}$.

15. (a) Show that the series $\frac{1}{2^3} - \frac{1}{3^3}(1+2) + \frac{1}{4^3}(1+2+3) - \frac{1}{5^3}(1+2+3+4) + \cdots$ converges.

Or

(b) Find the radius of convergence for the exponential series.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 600 words.

16. (a) Every pair of integers a and b has a common divisor d of the form $d = ax + by$ (x and y integers). Moreover, every common divisor of a and b divides this d.

Or

(b) State and prove triangle inequalities.

17. (a) (i) $(a_n) \to a,\ a_n \geq 0\ \forall n;\ a \neq 0 \Rightarrow \left(\sqrt{a_n}\right) \to \sqrt{a}$.

(ii) $(a_n) \to a, (b_n) \to b,\ b_n \neq 0 \forall n;\ b \neq 0 \Rightarrow \left(\frac{a_n}{b_n}\right) \to \frac{a}{b}$.

Or

(b) (i) Show that $\lim_{n\to\infty} a^{1/n} = 1$ where $a > 0$ is any real number.

(ii) Show that

$$\lim_{n\to\infty}\left[\frac{1}{\sqrt{2n^2+1}}+\frac{1}{\sqrt{2n^2+2}}+\cdots+\frac{1}{\sqrt{2n^2+n}}\right]=\frac{1}{\sqrt{2}}.$$

18. (a) State and prove Cauchy's first limit theorem.

Or

(b) State and prove Cauchy's general principle.

19. (a) Test the convergence of the series

$$\frac{1}{3}x+\frac{1.2}{3.5}x^2+\frac{1.2.3}{3.5.7}x^3+\cdots.$$

Or

(b) State and prove Gauss test.

20. (a) State and prove Leibnitz's test.

Or

(b) Let $\Sigma a_n x^n$ be the given power series. Let $\alpha = \lim\sup |a_n|^{1/n}$ and let $R = \frac{1}{\alpha}$ (if $\alpha = 0, R = \infty$ and if $\alpha = \infty, R = 0$). Then $\Sigma a_n x^n$ converges absolutely if $|x| < R$. If $|x| > R$, the series is not convergent.