(6 Pages) **Reg. No. : ......................................**

**Code No. : 20436 E** **Sub. Code : AMMA 31**

B.Sc. (CBCS) DEGREE EXAMINATION,
NOVEMBER 2021.

Third Semester

Mathematics — Core

SEQUENCES AND SERIES — I

(For those who joined in July 2020 onwards)

Time : Three hours Maximum : 75 marks

PART A — $(10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer :

1. If $x < y$, then for every $z$, $x + z$ ——— $y + z$

   (a) $<$ (b) $\leq$

   (c) $>$ (d) $\geq$

2. $[a, b] =$ ————

   (a) $a \leq x \leq b$ (b) $a \leq x < b$

   (c) $a < x \leq b$ (d) $a < x < b$

3. The range of the sequence $(1+(-1)^n)$ is ————

(a) $N$ (b) $z$

(c) $\{0, 1\}$ (d) $\{0, 2\}$

4. The value of $\lim_{n\to\infty} \frac{2n+1}{2n}$ is ————

(a) 0 (b) 1

(c) 2 (d) –1

5. The value of $\lim_{n\to\infty}\left(1+\frac{1}{2}+\cdots+\frac{1}{n}\right)$ is ————

(a) 0 (b) $e$

(c) 1 (d) $\infty$

6. Which of the following is a Cauchy sequence?

(a) $\left(\frac{1}{n}\right)$ (b) $(n)$

(c) $((-1)^n)$ (d) $(n^2)$

7. $(2^n)$ is a ———— sequence.

(a) convergent (b) divergent

(c) oscillating (d) none of these

8. The geometric series $\Sigma r^n$ converges if ————

(a) $0 < r < 1$ (b) $0 \le r \le 1$

(c) $0 \le r < 1$ (d) $r \ge 1$

9. If $\sum_{n=1}^{\infty} a_n$ converges to $s$ then ————

(a) $\lim_{n\to\infty} a_n = s$ (b) $\lim_{n\to\infty} a_n = 0$

(c) $\lim_{n\to\infty} a_n = a$ (d) $\lim_{n\to\infty} a_n = 1$

10. If $a_n = \frac{2^n n!}{n^n}$ then $\lim_{n\to\infty} \frac{a_n}{a_{n+1}}$ is ————

(a) $2e$ (b) $e$

(c) $\frac{1}{e}$ (d) $\frac{e}{2}$

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 250 words.

11. (a) Let $a$ and $b$ be real numbers, $a$ and $b$ such that $a \le b + \in$, for every $\in > 0$. Prove $a \le b$.

Or

(b) Define least upper bound and given an example.

12. (a) Examine the convergence of the sequence $((-1)^n)$.

Or

(b) If $(a_n) \to a$ and $a_n \geq 0$ for all $n$ prove $a \geq 0$.

13. (a) Show that if $|r| < 1$ then $(nr^n) \to 0$.

Or

(b) Prove that A sequence $(a_n)$ in $R$ is convergent iff it is a cauchy sequence.

14. (a) Show that $\sum\left(\frac{1}{n}\right)$ is not convergent.

Or

(b) State and prove Raabe's test.

15. (a) Show that the series $\sum(-1)^{n+1}\frac{n}{3n-2}$ oscillates.

Or

(b) Find the radius of convergence of the logarithmic series

$$x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots + (-1)^{n-1}\frac{x^n}{n} + \cdots$$

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions choosing either (a) or (b), answer should not exceed 600 words.

16. (a) State and prove the triangular inequality.

Or

(b) State and prove Cauchy – Schwarz inequality.

17. (a) Prove that a sequence cannot converge to two different limits.

Or

(b) Show that $\lim_{n\to\infty} \frac{3n^2+2n+5}{6n^2+4n+7} = \frac{1}{2}$.

18. (a) State and prove Cauchy's first limit theorem.

Or

(b) State and prove Cesaro's theorem.

19. (a) State and prove comparison test.

Or

(b) Prove that the harmonic series $\sum \frac{1}{n^p}$ converges if $p > 1$ and diverges if $p \le 1$.

20. (a) Prove that any absolutely convergent series is convergent.

Or

(b) State and prove Abel's theorem.