(7 pages) **Reg. No. :** ....

Code No. : 7753 **Sub. Code : WMAM 13**

M.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2023.

First Semester

Mathematics — Core

ORDINARY DIFFERENTIAL EQUATIONS

(For those who joined in July 2023 onwards)

Time : Three hours Maximum : 75 marks

PART A — (15 × 1 = 15 marks)

Answer ALL questions.

Choose the correct answer :

1. Two functions φ_1, φ_2 defined on an interval I are said to be ———— on I if there exists two constants c_1, c_2 not both zero such that $c_1\varphi_1(x)+c_2\varphi_2(x)=0$ for all x in I.
   (a) linearly independent
   (b) linearly dependent
   (c) both (a) and (b)
   (d) neither (a) nor (b)

2. The value of the Wronskian is ———— if $\varphi_1 = x^2$ and $\varphi_2 = 5x^2$.
   (a) 1 (b) 0
   (c) –1 (d) ∞

3. The characteristic polynomial of $y''+w^2y=0$ is ————.
   (a) $r^2+w^2=0$ (b) r^2+r
   (c) r^2+w^2r (d) r^2+r+w^2

4. The roots of the characteristic polynomial of the equation $y'''-3y'+2y=0$
   (a) 1, 1, –1 (b) 1, 1, 2
   (c) 1, 1, –2 (d) 1, –1, –2

5. The real valued solution of $y''-y=0$ is ————.
   (a) $c_1\cos x+c_2\sin x$ (b) $c_1e^x+c_2e^{-x}$
   (c) c_1+c_2x (d) $c_1x+c_2x^{-1}$

6. The particular solution of the equation $y''+4y=\cos x$ is ————.
   (a) $\frac{1}{3}\sin x$ (b) $\frac{1}{3}x$
   (c) $\frac{1}{3}$ (d) $\frac{1}{3}\cos x$

7. The linear differential equation $L(y)=b(x)$ is said to be homogeneous if $b(x)$ ———————.

(a) $\neq 0$ (b) $= 1$

(c) $= 0$ (d) > 1

8. The value of $P_n(-1)$ is ———————.

(a) $(-1)^n$ (b) 1

(c) 0 (d) 1^n

9. The value of the Legendre polynomial $P_2(x)$ is ———————.

(a) 1 (b) x

(c) x^2 (d) $\frac{3}{2}x^2-\frac{1}{2}$

10. The singular point and its nature of the equation $x^2y''+(x+x^2)y'-y=0$ is

(a) $x=0$, regular (b) $x=1$, regular

(c) $x=0$, irregular (d) $x=1$, irregular

11. The origin $x_0=0$ is ——————— for the equation $x^2y''-y'-\frac{3}{4}y=0$.

(a) singular point (b) regular singular

(c) irregular (d) analytic

12. Bessel equation has the ——————— as a regular singular point.

(a) Origin (b) $x=1$

(c) $x=\alpha$ (d) $x=-\alpha$

13. The solution of $y'=y^2$ with $\varphi(1)=-1$ is ———————.

(a) $-\frac{1}{x}$ (b) x

(c) x^2 (d) 0

14. The Lipschitz constant for the function $f(x, y)=x^2\cos^2 y+y\sin^2 x$ on $s:|x|\leq 1,\ |y|<\infty$ is ———————.

(a) 2 (b) 1

(c) 3 (d) -1

15. The equation $2xydx+(x^2+3y^2)dy=0$ is ———————.

(a) not exact (b) exact

(c) neither (a) nor (b) (d) both (a) and (b)

PART B — (5 × 4 = 20 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Compute the solution of the initial value problem $y''-2y'-3y=0$; $y(0)=0$, $y'(0)=1$.

Or

(b) Verify whether the functions φ_1, φ_2 given by $\varphi_1(x)=\sin x$, $\varphi_2(x)=e^{ix}$ are linearly independent or not.

17. (a) Find the solution of $y''' - y' = x$.

Or

(b) State and prove uniqueness theorem.

18. (a) Verify that $\varphi_1(x) = x$ satisfy the equation $x^2y'' - xy' + y = 0$.

Or

(b) Show that there exist n linearly independent solutions of $L(Y) = 0$ on I.

19. (a) Compute the indicial equation and the roots of $x^2y'' + (x + x^2)y' - y = 0$.

Or

(b) Prove that $J_0'(x) = -J_1(x)$.

20. (a) Give an example of a function satisfying Lipschitz condition.

Or

(b) Check the exactness of the equation $y' = \dfrac{3x^2 - 2xy}{x^2 - 2y}$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

21. (a) Find all solutions of $y'' - 7y' + 6y = \sin x$.

Or

(b) State and prove existence theorem for second order equations.

22. (a) Compute the solutions ψ of $y''' + y'' + y' + y = 1$.

Or

(b) Let φ be any solution of $L(y) = y^{(n)} + a_1 y^{(n-1)} + \dots a_n y = 0$ on I containing x_0. Then prove that for all x in I

$\|\varphi(x_0)\| e^{-k|x-x_0|} \le \|\varphi(x)\| \le \|\varphi(x_0)\| e^{k|x-x_0|}$ where $k = 1 + |a_1| + \dots + |a_n|$.

23. (a) Verify that the function $\varphi_1(x) = e^x (x > 0)$ satisfy the equation $xy'' - (x+1)y' + y = 0$ and find a second independent solution.

Or

(b) If $\varphi_1 \dots \varphi_n$ are n solutions of $L(y) = 0$ on I, prove that they are linearly independent if and only if $W(\varphi_1 \dots \varphi_n) \ne 0$ for all x in I.

24. (a) Find a solution φ of the form $\varphi(x) = |x-1|^r \sum_0^\infty c_k (x-1)^k$ for the Legendre's equation $(1-x^2)y'' - 2xy' + \alpha(\alpha+1)y = 0$.

Or

(b) Solve the Euler equation of nth order $x^n y^{(n)} + a_1 x^{n-1} y^{(n-1)} + \ldots + a_n y = 0$.

25. (a) Let M and N be two real valued functions which have continuous partial derivatives on $R : |x - x_0| \le a$, $|y - y_0| \le b$. Then prove that the equation $M(x, y) + N(x, y)y' = 0$ is exact if and only if $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ in R.

Or

(b) Consider the initial value problem $y' = 3y + 1$, $y(0) = 2$. Compute the first four approximations $\varphi_0, \varphi_1, \varphi_2, \varphi_3$. Compute the solution by direct method and compare.

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