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Reg. No. : .....

Code No. : 10295 E Sub. Code : AMMA 62

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2023.

Sixth Semester

Mathematics — Core

GRAPH THEORY

(For those who joined in July 2020 only)

Time : Three hours

Maximum : 75 marks

PART A — ( $10 \times 1 = 10$  marks)

Answer ALL questions.

Choose the correct answer :

1. The Independence number of  $K_p$  is \_\_\_\_\_

- (a) 2                      (b)  $p$   
(c)  $p-1$                 (d) 1

2.  $K_{m,n} =$  \_\_\_\_\_

- (a)  $K_m + K_n$             (b)  $K_m \cup K_n$   
(c)  $\overline{K_m} + \overline{K_n}$             (d)  $K_m \cap K_n$

3. Petersen graph is \_\_\_\_\_ graph

- (a) Eulerian  
(b) Hamiltonian  
(c) Eulerian, Non-Hamiltonian  
(d) Non Eulerian, Non-Hamiltonian

4. If  $e$  is a bridge of a graph  $G$ , then  $w(G-e) =$  \_\_\_\_\_

- (a)  $w(G)$                       (b)  $w(G)+1$   
(c)  $w(G)-1$                   (d)  $w(G)-2$

5. (A) Every edge of a tree is a bridge

(B) A block has no cut vertex

- (a) (A), (B) are correct  
(b) (A) is correct, (B) is wrong  
(c) (A) is wrong, (B) is correct  
(d) (A), (B) are wrong

6. The chromatic number of a tree is \_\_\_\_\_

- (a) 1                              (b) 2  
(c) 4                              (d) 0

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7. The number of faces in  $K_4$  is \_\_\_\_\_
- (a) 4 (b) 2  
(c) 6 (d) 5
8. The thickness of  $K_9$ ,  $\theta(K_9) =$  \_\_\_\_\_
- (a) 1 (b) 2  
(c) 3 (d) 4
9. A digraph on  $p$  vertices is functional if the out degree of every vertex is \_\_\_\_\_
- (a) 1 (b) 2  
(c)  $p$  (d)  $p-1$
10. (A) Strongly connected  $\Rightarrow$  weakly connected  
(B) Weakly connected  $\Rightarrow$  unilaterally connected
- (a) (A), (B) are correct  
(b) (A) is correct, (B) is wrong  
(c) (A) is wrong, (B) is correct  
(d) (A), (B) are wrong

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PART B — ( $5 \times 5 = 25$  marks)

Answer ALL questions choosing either (a) or (b).

11. (a) Show that sum of degrees of vertices of a graph is twice the number of edges.  
Or  
(b) If  $G$  is a  $(p, q)$  graph, show that  $\delta \leq \frac{2q}{p} \leq \Delta$ .
12. (a) If  $\delta \geq K$  show that the graph  $G$  has a path of length  $K$ .  
Or  
(b) Show that a closed walk of odd length contains a cycle.
13. (a) Prove that every connected graph has a spanning tree.  
Or  
(b) Prove that  $G$  is a tree iff  $G$  is connected and every line of  $G$  is a bridge.
14. (a) Prove that  $K_5$  is non-planar.  
Or  
(b) Prove that every planar graph  $G$  with  $p \geq 3$  vertices has at least 3 points of degree less than 6.

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[P.T.O.]





15. (a) In a digraph  $D$ , show that the sum of indegrees of all the vertices equals sum of their out degrees, each being equal to the number of arcs.

Or

- (b) If two digraphs are isomorphic, prove that corresponding points have the same degree pair.

PART C — ( $5 \times 8 = 40$  marks)

Answer ALL questions choosing either (a) or (b).

16. (a) Show that the maximum number of lines among all  $p$  point graphs with no triangles is

$$\left[ \frac{p^2}{4} \right].$$

Or

- (b) (i) Prove that, every graph is an intersection graph.  
(ii) A  $(p, q)$  graph has  $t$  vertices of degree  $m$  and all other vertices are of degree  $n$ . Show that  $(m - n)t + pn = 2q$ .

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17. (a) Prove that an edge  $x$  of a connected graph  $G$  is a bridge iff  $x$  is not on any cycle of  $G$ .

Or

- (b) State and prove the necessary and sufficient condition for a partition of an even number to be graphical.

18. (a) Prove that a  $(p, q)$  graph  $G$  is a tree iff  $G$  is acyclic and  $p = q + 1$ .

Or

- (b) If  $G$  is a plane  $(p, q)$  graph in which every face is an  $n$ -cycle, prove that  $q = \frac{n(p-2)}{n-2}$ .

19. (a) State and prove Dirac's theorem.

Or

- (b) Prove that every uniquely  $n$ -colourable graph is  $(n-1)$  connected.

20. (a) Prove that a weak digraph  $D$  is Eulerian iff every vertex of  $D$  has equal indegree and out degree.

Or

- (b) Find the chromatic polynomial of the graph with partition  $(3, 3, 3, 3, 2)$ .

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