(6 pages)

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B.Sc. (CBCS) DEGREE EXAMINATION,
NOVEMBER 2021.

Fifth Semester

Mathematics — Core

REAL ANALYSIS — II

(For those who joined in July 2017-2019)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer.

1. (M, d) is a discrete metric space. $a \in M$, then $B(a, 1) =$

   (a) ϕ (b) M

   (c) $\{a\}$ (d) $\{a, 1\}$

2. Which of the following is open in R with usual metric?

   (a) N (b) Z

   (c) Q (d) R

3. If a subset A of a metric space M is nowhere dense in M then

(a) $\text{int}\, A = \phi$ (b) $\text{int}\, \overline{A} = \phi$

(c) $\overline{A} = \phi$ (d) $\overline{A} = M$

4. $D(Q) =$

(a) ϕ (b) Z

(c) Q (d) R

5. If f is a continuous function then

(a) $f(\overline{A}) = \overline{f(A)}$ (b) $f(\overline{A}) \supseteq \overline{f(A)}$

(c) $f(\overline{A}) \subseteq \overline{f(A)}$ (d) $f(A) = f(\overline{A})$

6. Which of the following is not a type F_σ?

(a) Q (b) $R - Q$

(c) Z (d) N

7. Which of the following is connected in R with usual metric?

(a) Q (b) R

(c) $[1, 2] \cup [3, 4]$ (d) Z

8. In R with discrete metric which is compact?

(a) $\{1, 2\}$ (b) $(1, 2)$

(c) $[1, 2]$ (d) R

9. $\int_0^3 [x]dx =$

(a) 0 (b) 2

(c) 4 (d) 3

10. If $f(x)$ is bounded and integrable in $\left[0, \frac{\pi}{2}\right]$, then

$\lim_{n\to\alpha} \int_0^{\frac{\pi}{2}} f(x)\sin nx dx$

(a) $= 0$ (b) $= 1$

(c) > 0 (d) > 1

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) In a metric space (M, d), prove that $|d(x, z) - d(y, z)| \leq d(x, y)$ for all $x, y, z \in M$.

Or

(b) Let (M, d) be a metric space. If $A \subseteq M$, prove that $\text{int} A =$ Union of all open sets contained in A.

12. (a) Let (M, d) be a metric space. If $A, B \subseteq M$ prove that $\overline{A \cup B} = \overline{A} \cup \overline{B}$.

Or

(b) Prove that any discrete metric space is complete.

13. (a) Show that the function $f : R \to R$ defined by $f(x) = \sin x$ is uniformly continuous on R.

Or

(b) If $f : M \to R$ is a continuous function defined on a metric space M prove that $\{x \in M / f(x) \geq 0\}$ is a closed set.

14. (a) If A and B are connected subsets of a metrix space (M, d) and $A \cap B \neq \phi$ prove that $A \cup B$ is connected.

Or

(b) Prove that a closed subset of a compact metric space is also compact.

15. (a) Prove that the function f defined by

$$f(x) = \begin{cases} 0, \text{ if } x \text{ is rational,} \\ 1, \text{ if } x \text{ is irrational} \end{cases}$$

is not integrable on any interval.

Or

(b) State and prove Cauchy's Mean value theorem.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Let (M, d) be a metric space. Define $\rho(x, y) = 2d(x, y)$. Prove that d and ρ are equivalent metrices.

Or

(b) Show that any open subset of R can be written as the union of a countable number of mutually disjoint open intervals.

17. (a) Prove that in any metric space every closed sphere is a closed set.

Or

(b) State and prove Cantor's intersection theorem.

18. (a) Prove that $f : M_1 \to M_2$ is continuous iff inverse image of every open set is open.

Or

(b) Prove that $f : R \to R$ is continuous at $a \in R$ iff $w(f, a) = 0$.

19. (a) Prove that a subspace A of R is connected if and only if A is an interval.

Or

(b) State and prove Heine Borel theorem.

20. (a) State and prove Rolle's Theorem.

Or

(b) (i) If E is measurable prove that for all $\varepsilon > 0$ there exists an open set $0 \supseteq E$ such that $m^{*}(0 - E) \leq E$.

(ii) State and prove Lagrange's mean value theorem.
