(7 pages) **Reg. No. :** ....

Code No. : 9071 **Sub. Code : PMAM 13**

M.Sc. (CBCS) DEGREE EXAMINATION,
NOVEMBER 2017.

First Semester

Mathematics

ANALYTIC NUMBER THEORY

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

SECTION A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. If $(a, b) = (a, c) = 1$, then $(a, bc) =$

(a) a (b) b

(c) c (d) 1

2. If $(a, b) = 1$, then $(a+b, a^2 - ab + b^2) =$

(a) either 1 or 2 (b) either 1 or 3

(c) 2 (d) 4

3. $\phi(2^\alpha) =$

(a) 1 (b) 2^α

(c) $2^{\alpha-1}$ (d) $2^{\alpha+1}$

4. $u(n) =$

(a) n (b) 1

(c) $\log n$ (d) $\sum_{p|n} \log p$

5. The Mangoldt function $\wedge(n)$ is

(a) multiplicative

(b) not multiplicative

(c) completely multiplicative

(d) multiplicative but not completely multiplicative

6. $\lambda^{-1}(n) =$

(a) 1 (b) $|\mu(n)|$

(c) -1 (d) ± 1

7. $\lim_{x\to\infty}\left(\sum_{n\le x}\frac{1}{n}-\log x\right)=$

(a) Euler's constant

(b) Riemann Zeta function

(c) $O\left(\frac{1}{x}\right)$

(d) $O\left(x^{1-s}\right)$

8. $\sum_{n>x}\frac{1}{n^s}=$ ————, (here $S>1$).

(a) $O\left(\frac{1}{x}\right)$ (b) $O\left(x^s\right)$

(c) $O\left(x^{1-s}\right)$ (d) $O\left(x^{s-1}\right)$

9. $\sum_{n\le x}\mu(n)\left[\frac{x}{n}\right]=$

(a) 0 (b) 1

(c) $\log x$ (d) $\log[x]!$

10. The smallest integer $x\ge 0$ for which x^2+x+41 is composite is

(a) 39 (b) 40

(c) 41 (d) 42

SECTION B — ($5\times 5=25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that n^4+4 is composite if $n>1$.

Or

(b) Prove that every integer $n>1$ is either a prime number or a product of prime numbers.

12. (a) If $n\ge 1$, prove that $\sum_{d|n}\phi(d)=n$.

Or

(b) Prove that if $a\mid b$ then $\phi(a)\mid\phi(b)$.

13. (a) If f and g are multiplicative functions then, prove that $f*g$ is also multiplicative.

Or

[P.T.O.]

(b) Give f is multiplicative. Prove that f is completely multiplicative if and only if, for all primes p and all integers $a \geq 2$, $f^{-1}(p^a) = 0$.

14. (a) Prove that if $x \geq 1$, $s > 0, s \neq 1$ then $\sum_{n \leq x} \frac{1}{n^s} = \frac{x^{1-s}}{1-s} + \varsigma(s) + O(x^{-s})$.

Or

(b) Prove that, if $x > 1$, $\sum_{n \leq x} \phi(n) = \frac{3x^2}{\pi^2} + O(x \log x)$.

15. (a) For $x \geq 1$, show that $\left|\sum_{n \leq x} \frac{\mu(n)}{n}\right| \leq 1$, with equality holding only if $x < 2$.

Or

(b) Calculate the highest power of 10 that divides 1000!

SECTION C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) State and prove the fundamental theorem of Arithmetic.

Or

(b) Show that the infinite series $\sum_{n=1}^{\infty} \frac{1}{p_n}$ diverges.

17. (a) For $n \geq 1$, prove that $\phi(n) = n \prod_{p|n} \left(1 - \frac{1}{p}\right)$.

Or

(b) If $n \geq 1$, show that $\log n = \sum_{d|n} \wedge(d)$.

18. (a) State and prove Euler's summations formula.

Or

(b) Prove that the set of lattice points visible from origin has density $\frac{6}{\pi^2}$.

19. (a) State and prove Generalized Möbius inversion formula.

Or

(b) Given f is multiplicative. Show that f is completely multiplicative if and only if $f^{-1}(n) = \mu(n) f(n)$ for all $n \geq 1$.

20. (a) Show that the following are logically equivalent :

(i) $\lim_{x \to \infty} \frac{\pi(x) \log x}{x} = 1$;

(ii) $\lim_{x \to \infty} \frac{\theta(x)}{x} = 1$;

(iii) $\lim_{x \to \infty} \frac{\psi(x)}{x} = 1$.

Or

(b) Prove that for every integer $n \geq 2$, $\frac{n}{6 \log n} < \pi(n) < \frac{6n}{\log n}$.
