(7 pages) **Reg. No. :** .....

Code No. : 5373 **Sub. Code : ZMAM 25**

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2023.

Second Semester

Mathematics – Core

RESEARCH METHODOLOGY AND STATISTICS

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. Text books are example of _____ of information.

   (a) Primary sources (b) Secondary sources

   (c) Constant (d) Variable

2. The length of the abstract may be _____ words.

   (a) 100 (b) 200

   (c) 300 (d) 400

3. The marginal p.d.f. $f_2(x_2)$ of X_2 in discrete case is _____.

   (a) $\sum_{x_1} f(x_1 x_2)$ (b) $\sum_{x_2} f(x_1 x_2)$

   (c) $\sum_{x_1} f(x_1 x_1)$ (d) $\sum_{x_1} f(x_2 x_2)$

4. If the joint p.d.f. of the random variable $x_1, x_2, f(x_1, x_2) = x_1 + x_2, \quad 0 < x_1 < 1, \quad 0 < x_2 < 10$ elsewhere then the marginal p.d.f. of X_1 is _____.

   (a) $x_2 + 1$ (b) $x_2 + \frac{1}{2}$

   (c) $x_1 + \frac{1}{2}$ (d) $x_2 + 2$

5. The mean of the gamma distribution is _____.

   (a) $\alpha\beta$ (b) $\alpha\beta^2$

   (c) $\alpha 2\beta$ (d) $3\alpha\beta$

6. The variance of Chi-square distribution $\chi^2(r)$ is _____.

   (a) r (b) $2r$

   (c) $3r$ (d) r^2

7. A function of one or more random variables that does not depend upon any unknown parameters is called a

(a) statistic (b) parameter

(c) unit (d) variance

8. If F has an F-distribution with parameters r_1 and r_2 then $1/F$ has an F-distribution with parameters _____ and _____.

(a) $1/r_1, 1/r_2$ (b) r_2, r_1

(c) $1/r_2, 1/r_1$ (d) r_1, r_2

9. The mean of the random sample $x_1, x_2, \ldots x_n \quad n \geq 2$ is _____.

(a) $\frac{\sum x_i}{n}$ (b) $1/n$

(c) x_i/n (d) $X_i * n$

10. If $f(x_1, x_2) = \frac{x_1 x_2}{36}$, $x_1 = 1, 2, 3$, $x_2 = 1, 2, 3$ / 0 elsewhere then $\Pr(x_1 = 2, x_2 = 3)$ is _____.

(a) 0 (b) 1

(c) 1/6 (d) 1/2

PART B — (5 × 5 = 25 marks)

Answer ALL questions by choosing either (a) or (b).

11. (a) What are the types, methods and techniques used in research methodology?

Or

(b) State the any three research process in flow chart.

12. (a) The joint p.d.f. of the random variable X and Y is $f(x, y) = 6x^2 y$, $0 < x < 1, 0 < y < 1$ and zero elsewhere then find $\Pr(0 < x < 3/4, 1/3 < y < 2)$.

Or

(b) Let the joint p.d.f. of the random variables X_1 and X_2 be $f(x_1, x_2) = x_1 + x_2$, $0 < x_1 < 1, 0 < x_2 < 1$ and zero else where then prove that X_1 and X_2 are dependent.

13. (a) Let X be $\chi^2(10)$. Then find $\Pr(3.25 < X < 20.5)$.

Or

(b) If $(1 - 2t)^{-6}, t < 1/2$ is the m.g.f. of the random variable X then find $\Pr(X < 5.23)$.

14. (a) Let X have the p.d.f. $f(x)=1/3$, $x=1, 2, 3,$ zero else then find the p.d.f. of $Y=2X+1$.

Or

(b) Let X have the p.d.f. $f(x)=2x, 0<x<1$ zero then find the value of the jacobian if $Y=8x^3$ also the p.d.f. of Y.

15. (a) Let X_1 and X_2 be independent with normal distributions $N(\mu_1, \sigma_2^2)$ and $N(\mu_2, \sigma_2^2)$ respectively. Then for the random variable $Y=X_1-X_2$, find the p.d.f. $g(y)$ of Y.

Or

(b) Let the random variable X_1, X_2 have the same p.d.f. $f(x)=x/6$, $x=1, 2, 3$, 0 else. Then find $\Pr(X_1+X_2=3)$

PART C — (5 × 8 = 40 marks)

Answer ALL questions by choosing either (a) or (b).

16. (a) Write about bibliography and appendices.

Or

(b) Explain about the review of literature.

17. (a) Let X_1, X_2 and X_3 be three mutually independent random variables and let each have the p.d.f. $f(x)=2x, 0<x<1$, zero elsewhere. The joint p.d.f. of X_1, X_2 and X_3 is $f(x_1)f(x_2)f(x_3)=8x_1x_2x_3, 0<x_i<1, i=1,2,3$, zero elsewhere. Find the expected value of $5X_1X_2X_3+3X_2X_3^4$. Also find the p.d.f. of the random variable Y the maximum of X_1, X_2 and X_3.

Or

(b) Let X_1 and X_2 denote random variables that have the joint p.d.f. $f(x_1, x_2)$ and the marginal probability density functions $f_1(x_1)$ and $f_2(x_2)$ respectively. Let $M(t_1 t_2)$ be the m.g.f. of the distribution. Then prove that X_1 and X_2 are independent if and only if $M(t_1, t_2)=M(t_1, 0)M(0, t_2)$.

18. (a) Let X have a gamma distribution with $\alpha=r/2$, where r is a positive integer and $\beta>0$. If the random variable $Y=2X/\beta$ find the p.d.f. of Y.

Or

(b) Prove that if the random variable X is $N(\mu, \sigma^2)$, $\sigma^2>0$, then the random variable $V==(X-\mu)^2/\sigma^2$ is $\chi^2(1)$.

19. (a) Derive 't' distribution.

Or

(b) If F has F–distribution with parameters $r_1 = 5$ and $r_2 = 10$, find a and b so that $\Pr(F \le a) = 0.05$ and $\Pr(F \le b) = 0.95$ and $\Pr(a < F < b) = 0.90$.

20. (a) Prove that (i) $\overline{X}$ is $N(\mu, \sigma^2/n)$ (ii) ns^2/σ^2 is $\chi^2(n-1)$ (iii) $\overline{X}$ and S are independent.

Or

(b) Let $X_1, X_2, \ldots X_n$ be independent random variables having respectively, the normal distributions $N(\mu, \sigma_1^2)$, $N(\mu_2, \sigma_2^2)\ldots$ and $N(\mu_n, \sigma_n^2)$. Then prove that the random variable $Y = k_1X_1 + k_2X_2 + \ldots + k_nX_n$ where $k_1, k_2, \ldots, k_n$ are real constants, be normally distributed with mean $k_1\mu_1 + \ldots + k_n\mu_n$ and variance $k_1^2\sigma_1^2 + \ldots + kn^2\sigma_n^2$.
