(8 pages) **Reg. No. :**..

Code No. : 20590 E **Sub. Code : SEMA 6 B**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2021.

Sixth Semester

Mathematics

Major Elective — FUZZY MATHEMATICS

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer.

1. A fuzzy set A is called normal when $h(A)$ is

   (a) 0 (b) 1

   (c) < 1 (d) > 1

2. $S(A, B) =$ ___________.

   (a) $\frac{|A \cup B|}{|A|}$ (b) $\frac{|A \cap B|}{|A|}$

   (c) $\frac{|A \cup B|}{|B|}$ (d) $\frac{|A \cap B|}{|B|}$

3. Let $A, B \in \mathcal{F}(X)$ and $\alpha, \beta \in [0, 1]$. Then $\alpha \le \beta \Rightarrow$

(a) $\alpha_A \supseteq \beta_A$ (b) $\alpha_A \supseteq \beta^{+}{}_A$

(c) $\alpha^{+}{}_A \subseteq \beta_A$ (d) $\alpha_A \supseteq \beta_A$

4. Let $f : X \to Y$ be an arbitrary crisp function and $A \in \mathcal{F}(X)$, then

(a) $A \subset f^{-}(f(A))$ (b) $A \supset f^{-1}(f(A))$

(c) $A \subseteq f^{-1}(f(A))$ (d) $A \supseteq f^{-1}(f(A))$

5. For $W = \left(\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n}\right)$, h_W is the

(a) arithmetic mean (b) geometric mean

(c) harmonic mean (d) generalized mean

6. $u(a, b) = \min(1, a + b)$ is known as

(a) Standard Union (b) Algebraic Sum

(c) Bounded Sum (d) Drastic Union

7. The property $A \cdot (B + C) \subseteq A \cdot B + A \cdot C$ is known as

(a) associativity (b) distributivity

(c) subassociativity (d) subdistributivity

8. To qualify as a fuzzy number a fuzzy set A on $\mathbb{R}$ must be

(a) convex (b) not convex

(c) subnormal (d) normal

9. In a Linear Programming Problem, the matrix $A = \lfloor a_{ij} \rfloor,\ i \in N_m,\ j \in N_n$ is

(a) goal matrix (b) constraint matrix

(c) cost matrix (d) decision matrix

10. In multiperson decision making $S(x_i, x_j)$ = ___________.

(a) $\frac{N(x_i, x_j)}{n}$ (b) $\frac{N(x_j, x_i)}{n}$

(c) $\frac{n}{N(x_i, x_j)}$ (d) $\frac{n}{N(x_j, x_i)}$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Define :

(i) α-cut

(ii) Strong α-cut

(iii) Height of a fuzzy set A

(iv) Normal fuzzy set

(v) Subnormal fuzzy set.

Or

(b) Prove that : A fuzzy set A on $\mathbb{R}$ is convex $\Leftrightarrow$ $A(\lambda x_1 + (1-\lambda)x_2) \geq \min[A(x_1), A(x_2)]$ for all $x_1, x_2 \in \mathbb{R}$ and all $\lambda \in [0, 1]$ where min denotes the minimum operator.

12\. (a) Let $A, B \in \mathcal{F}(X)$. Then for all $\alpha \in [0, 1]$, prove that $\alpha(\overline{A}) = {}^{(1-\alpha)+}\overline{A}$.

Or

(b) Let $f: X \rightarrow Y$ be an arbitrary crisp function. Then prove that for any $A \in \mathcal{F}(X)$, f fuzzfied by the extension principle satisfies the equation $f(A) = \bigcup_{\alpha \in [0, 1]} f({}_{\alpha+}A)$.

13\. (a) Let $\langle i, u, c \rangle$ be a dual triple that satisfies the law of excluded middle and law of contradiction. Then, prove that $\langle i, u, c \rangle$ does not satisfy the distributive laws.

Or

(b) Prove that, the standard fuzzy intersection is the only idempotent t-norm.

14. (a) Explain Linguistic Variables.

Or

(b) Let MIN and MAX be binary operations on $\mathcal{R}$ defined by

$$\text{MIN}(A, B)(Z) = \underset{Z=\min(x,\, y)}{Sup} \min[A(x), B(y)],$$

$$\text{MAX}(A, B)(Z) = \underset{Z=\max(x,\, y)}{Sup} \min[A(x), B(y)]$$ for all $Z \in \mathbb{R}$. Then for any $A, B, C \in \mathcal{R}$ prove that,

$$\text{MIN}[\text{MIN}(A, B), C] = \text{MIN}[A, \text{MIN}(B, C)].$$

15. (a) Explain the method of solving the fuzzy linear programming problem defined by,

$$\max \sum_{j=1}^{n} C_j x_j$$

$$\text{s.t } \sum_{j=1}^{n} A_{ij} x_j \leq B_i \quad (i \in N_m)?$$

$$x_j \geq 0 \quad (j \in N_n)$$

where B_i and A_{ij} are fuzzy numbers.

Or

(b) Solve the following fuzzy linear programming problem.

$$\max \quad Z = 6x_1 + 5x_2$$

$$\text{S.t} \quad \langle 5, 3, 2 \rangle x_1 + \langle 6, 4, 2 \rangle x_2 \leq \langle 25, 6, 9 \rangle$$

$$\langle 5, 3, 2 \rangle x_1 + \langle 2, 1.5, 1 \rangle x_2 \leq \langle 13, 7, 14 \rangle$$

$$x_1, x_2 > 0$$

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Explain Interval Valued Fuzzy Sets. Also write down the advantages and disadvantages of Interval Valued Fuzzy Sets.

Or

(b) Write down all the fundamental properties of crisp set operations.

17. (a) State and prove Second Decomposition Theorem.

Or

(b) Let $f : X \to Y$ be an arbitrary crisp function. Then for any $A \in \mathcal{F}(X)$ and all $\alpha \in [0, 1]$, prove that the following properties of f fuzzified by the extension principle hold :

(i) ${}^{\alpha+}[f(A)] = f\left({}^{\alpha+}A\right)$

(ii) ${}^{\alpha}[f(A)] \supseteq f\left({}^{\alpha}A\right)$.

18. (a) State and prove first characterization theorem of fuzzy complements.

Or

(b) For all $a, b \in [0, 1]$, prove that $\max(a, b) \leq u_w(a, b) \leq u_{\max}(a, b)$.

19. (a) Calculate the following :

(i) $[2, 5] + [1, 3]$

(ii) $[2, 5] - [1, 3]$

(iii) $[1, 1] \cdot [-2, -0.5]$

(iv) $[1, 1] / [-2, -0.5]$.

Or

(b) Let $* \in \{+, -, \cdot, /\}$ and A, B be continuous fuzzy numbers defined by $(A * B)(Z) = \underset{Z=x*y}{Sup} \min[A(x), B(y)]$ for all $Z \in \mathbb{R}$, show that fuzzy set $A * B$ is a continuous fuzzy set.

20. (a) Explain Multiperson Decision Making.

Or

(b) Solve the following fuzzy linear programming problem.

Max $Z = .4x_1 + .3x_2$

S.t $x_1 + x_2 \le B_1$

$2x_1 + x_2 \le B_2$

$x_1, x_2 \ge 0$

Where B_1 is defined by

$$B_1(x) = \begin{cases} 1 & \text{when } x \le 400 \\ \frac{(500 - x)}{100} & \text{when } 400 < x \le 500 \\ 0 & \text{when } 500 < x \end{cases}$$

and B_2 is defined by

$$B_2(x) = \begin{cases} 1 & \text{when } x \le 500 \\ \frac{(600 - x)}{100} & \text{when } 500 < x \le 600 \\ 0 & \text{when } 600 < x \end{cases}$$