(7 pages) **Reg. No. :** ....

Code No. : 6351 **Sub. Code :HMAM 11**

M.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2013.

First Semester

Mathematics – Main

ALGEBRA — I

(For those who joined in July 2012 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. Suppose $p = G \to \overline{G}$ is a group homomorphism then which one of the following is not true
   (a) $\varphi(e) = \overline{e}$ where e and $\overline{e}$ are unit elements of G and $\overline{G}$ respectively
   (b) $\varphi(x^{-1}) = (\varphi(x))^{-1}$ for all $x \in G$
   (c) The kernel of φ is a normal subgroup of G
   (d) The kernel of φ is a abelian subgroup of G

2. Suppose $A(G)$ is the set of all automorphisms of G then which one of the following is not true
   (a) If G is a group the $A(G)$ also group
   (b) If G is an abelian group then $A(G)$ is also a group
   (c) If G is a cyclic group then $A(G)$ is also a group
   (d) If G is a non abelian group then $A(G)$ is not a group

3. Which one of the following is false?
   (a) (1,2,3) (1,3,2) = &C
   (b) (1,2,3) (2,3) =(1,3)
   (c) $(1,2,3,4)^{-1}$ = (1,2,3,4)
   (d) (1,2,3) (5,6,4,1,18) =(2,3,8,1,6,4,7,5)

4. Which one of the following is true
   (a) If $10/0(G)$ then there is a subgroup of order 10.
   (b) If $a \in Z$ the $N(a) = G$.
   (c) Z is a cyclic subgroup of G.
   (d) If $o(G) = 49$ then G need not e abelian

5. The number of Non-isomorphic abelion group of order 3^5 is

(a) 1 (b) 3

(c) 5 (d) 6

6. Which one of the following is false?

(a) Suppose that G is the internal direct product of $N_1, N_2, \ldots N_R$, if $a \in N_i$ & $b \in N_j$, $i \neq j$ then $ab = ba$.

(b) If A and B are groups, then $A \times B$ is isomorphic to $B \times A$

(c) If G is a group and $T = G \times G$ then $D = \{(g.g) \in G \times G / g \in G\}$ is a group

(d) IF A and B are cyclic groups. Then $A \times B$ in cyclic.

7. Which one of the following is false

(a) $[a,b]+[c,d]=[ab+bc,bd]$

(b) $[a,b]^{-1}=[b,a]$

(c) $[0,0]$ is the zero element in the addition

(d) If $[a,b]=[a',b']$ and $[c,d]=[c',d]$ then $[a,b][c,d] = [a',b'][c',d']$

8. If R is an Eulidean ring then which one of the following is false

(a) If $A = (a_0)$ is a maximal ideal of R Then a_0 is a prime element

(b) If a_0 is a prime element then $A = (a_0)$ is a prime element

(c) R is a unique factorization domain

(d) Every ideal in R is a prime ideal

9. If $f(x)$ and $g(x)$ are two non zero elements in $F(x)$ then which one of the following is false.

(a) $\deg(f(x).g(x)) = \deg(f(x)) + \deg(g(x))$

(b) $\deg(f(x)) \leq \deg(f(x).g(x))$

(c) $\deg(f(x)+g(x)) = \deg(f(x)) + \deg(g(x))$

(d) $F(x)$ is a principals ideal ring.

10. Which one of the following is false?

(a) If $f(x)$ and (x) are primitive polynomial then so in $f(x).g(x)$

(b) If R is an integral domain so in $R[x]$

(c) If R is a UFD then so its $R[x]$

(d) If R is a UFD and $a \in R$ and a/bc then either a/b or a/c

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Let G be a group, T an automorphism of G. N is a normal subgroup of G. Prove that $T(N)$ is a normal subgroup of G.

Or

(b) If H is subgroup of G, the prove that $W = \bigcap_{g \in G} gHg^{-1}$ is a normal subgroup of G.

12. (a) If G is a finite group, then prove that $C_a = o(G)/o(N(a))$.

Or

(b) If $o(G) = P^n$ where p is a prime number, then prove that $Z(G) \neq (e)$.

13. (a) Suppose that G is the internal direct product of $N_1, N_2 .... N_k$. The prove that for $i \neq j$, $N_i \cap N_j = (e)$ and if $a \in N_i, b \in N_j$ then $ab = ba$.

Or

(b) Let G be a group and suppose that G is the internal direct product $N_1, N_2 .... N_k$. Let $T = N_1 \times N_2 \times ..... \times N_k$. The prove that G and T are isomorphic.

14. (a) Let R be a Eulidean ring and A is an ideal of R. Then prove that A is a principal ideal.

Or

(b) State and prove the Fermat's theorem.

15. (a) State and prove the Division algorithm theorem.

Or

(b) State and prove the Eisenstein criterion.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) State and prove the Cauchy's theorem of abelian groups.

Or

(b) If G is a group, H a subgroup of G, and S in the set of all right coset of H in G, then prove that there is a homomorphism θ of G in $A(s)$ and the kernel of θ largest normal subgroup of G which is contained in H.

17. (a) State and prove the Cauchy's theorem.

Or

(b) State and prove the sylow's third part.

18. (a) Prove that two abelian groups of order p^n are isomorphic if and only if they have the same invariant.

Or

(b) Prove that every finite abelian group is the direct product of cyclic groups.

19. (a) Prove that every integral domain can be imbedded in a field.

Or

(b) Prove that $J[i]$, the set of all Gaussian integers forms an Euclidean ring.

20. (a) Let F be the field of real numbers. Prove that $F[x]/(x^2+1)$ is a field isomorphic to the field of complex numbers.

Or

(b) Prove that a principal ideal ring is a unique factorization domain.

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