

Reg. No. : .....

Code No. : 9023

Sub. Code : PMAC 11

M.Phil. DEGREE EXAMINATION,  
NOVEMBER 2022

First Semester

Mathematics

RESEARCH AND TEACHING METHODOLOGY

(For those who joined in July 2018 – 2019 onwards)

Time : Three hours

Maximum : 75 marks

PART A — ( $5 \times 5 = 25$  marks)

Answer ALL questions, choosing either (a) or (b).

1. (a) Prove that  $x \in \mathbb{R} \Leftrightarrow 1 - xy$  is a unit in  $A$  for all  $y \in A$ .

Or

- (b) Prove that the radical of an ideal  $a$  is the intersection of the prime ideals which contain  $a$ .

2. (a) Show that if  $\mathcal{R}$  is the nilradical of  $A$ , the nilradical of  $S^{-1}A$  is  $S^{-1}\mathcal{R}$ .

Or

- (b) Prove that let  $q$  be a primary ideal in a ring  $A$ , then  $r(q)$  is smallest prime ideal containing  $q$ .



3. (a) Show that let  $A \subseteq B$  be rings and let  $C$  be the integral closure of  $A$  in  $B$ , then  $C$  is integrally closed in  $B$ .

Or

- (b) Prove that let  $A \subseteq B$  be rings,  $B$  integral over  $A$ , and let  $p$  be a prime ideal of  $A$ , then there exists a prime ideal  $q$  of  $B$  such that  $q \cap A = p$ .
4. (a) Prove that if  $A$  is Noetherian and  $p$  is a prime ideal of  $A$  then  $A_p$  is Noetherian.

Or

- (b) Prove that in a Noetherian ring the nilradical is nilpotent.
5. (a) Write short notes on Lecture Method.

Or

- (b) What do you mean by Seminar?

PART B — (5 × 10 = 50 marks)

Answer ALL questions, choosing either (a) or (b).

6. (a) Prove that  $M$  is a finitely generated  $A$ -module  $\Leftrightarrow M$  is isomorphic to a quotient of  $A^n$  for some integer  $n > 0$ .

Or

- (b) State and prove Nakayama's lemma.

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7. (a) Prove that let  $M$  be a finitely generated  $A$ -module,  $S$  a multiplicatively closed subset of  $A$ , then  $S^{-1}(Ann(M)) = Ann(S^{-1}M)$ .

Or

- (b) State and prove 2<sup>nd</sup> uniqueness theorem.
8. (a) State and prove Going-up theorem.

Or

- (b) Show that :
- (i)  $B$  is a local ring
- (ii) If  $B'$  is a ring such that  $B \subseteq B' \subseteq K$ , then  $B'$  is a valuation ring of  $K$
- (iii)  $B$  is integrally closed (in  $K$ ).
9. (a) Prove that let  $B$  be a finitely generated  $A$ -algebra if  $A$  is Noetherian then  $S_0$  is  $B$ .

Or

- (b) State and prove structure theorem for Artin rings.
10. (a) Explain about Integrating ICT in Teaching.

Or

- (b) Explain about Evaluation.

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