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Reg. No. :

Code No. : 6067

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M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2011.

Fourth Semester

Mathematics

COMPLEX ANALYSIS

(For those who joined in July 2008 and afterwards)

Time : Three hours

Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL the questions.

1. Write down the Cauchy-Riemann differential equations.
2. When do we say that the geometric series $1 + z + z^2 + \dots + z^n + \dots$ is convergent?
3. When do we say that an analytic function is said to be degenerates?

4. Whether the points $z \rightarrow \bar{z}$ are symmetric with respect to the real axis or not?

5. Find $\int_C \frac{dz}{z-a}$ where C is the circle with center a and radius r .

6. Prove or disprove : $f(z) = \frac{1}{z(z-1)^2}$ is a meromorphic function.

7. When do we say that two chains should be considered identical?

8. If Γ is the image cycle of a cycle γ and Γ lies in a disk which does not contain the origin, then find $n(\Gamma, 0)$.

9. Write down the polar coordinates form of the Laplace equation.

10. Find the Laurent's series for $f(z) = \frac{1}{z(z-1)}$ about $z = 0$.

PART B — (5 × 5 = 25 marks)

Answer ALL the questions, choosing either (a) or (b).

11. (a) If a function is analytic, show that it is independent of $\bar{z}$.

Or

- (b) State and prove Lucas's theorem.

12. (a) Prove that $\omega = z^{\frac{1}{2}}$ is continuous in the complement of the negative real axis $z \leq 0$.

Or

- (b) Show that the cross ratio is real if and only if the four points lie on a circle or a straight line.

13. (a) Compute $\int_{|z|=2} \frac{dz}{z^2-1}$ for the positive sense of the circle.

Or

- (b) State and prove Morera's theorem.

14. (a) Prove that $pdx + qdy$ is locally exact in Ω if and only if $\int_{\partial R} pdx + qdy = 0$ for every rectangle $R \subset \Omega$ with sides parallel to the axes.

Or

- (b) Show that single valued analytic branch of $\log z$ can be defined in any simple connected region which does not contain the origin.

15. (a) Prove that the arithmetic mean of a harmonic function over concentric circles $|z| = r$ is a linear function of $\log r$.

Or

- (b) What is the coefficient of z^7 in the Taylor's development of $\tan z$?

PART C — (5 × 8 = 40 marks)

Answer ALL the questions, choosing either (a) or (b).

16. (a) State and prove the necessary and sufficient condition for a function to be analytic.

Or

- (b) Discuss completely the convergence and uniform convergence of the sequence $\{nz^n\}_1^\infty$.

17. (a) Suppose that the coefficients of the transformation $S_z = \frac{az+b}{cz+d}$ are normalized by $ad-bc=1$. Show that S is elliptic if and only if $-2 < a+d < 2$.

Or

- (b) Discuss the transformation $w = z^2$
18. (a) State and prove Cauchy's theorem for a rectangle.

Or

- (b) (i) State and prove the fundamental theorem of algebra.
- (ii) Compute $\int_{|z|=1} e^z z^{-n} dz$.
19. (a) Show that the bounded regions determined by a closed curve are simply connected while the unbounded region is doubly connected.

Or

- (b) Prove that $\int_0^\pi \log \sin \theta d\theta = -\pi \log 2$.

20. (a) Let u_1 and u_2 be harmonic in a region Ω . If $u_1 = 1$, $u_2 = u$, then prove that $\int_\gamma \left(u_1 \frac{\partial u_2}{\partial n} - u_2 \frac{\partial u_1}{\partial n} \right) |dz| = 0$ for every cycle γ which is homologous to zero in Ω .

Or

- (b) Show that every function f which is analytic in a symmetric region Ω can be written in the form $f_1 + if_2$ where f_1 and f_2 are analytic in Ω and real on the real axis.