20. (a) Derive the equation of a common catenary.

Or

(b) A string hangs under gravity and is so loaded that the weight on each element of it is proportional to the horizontal projection of that element; show that it will hang in the form of a parabola.

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Reg. No. : .....

Code No. : 20424 E **Sub. Code : CMMA 53**

B.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2023.

Fifth Semester

Mathematics — Core

STATICS

(For those who joined in July 2021–2022)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer.

1. The resultant of two equal forces P, P at an angle α is ———————

(a) $2P\cos\alpha$ (b) $2P\sin\alpha$

(c) $2P\sin\frac{\alpha}{2}$ (d) $2P\cos\frac{\alpha}{2}$

2. Forces acting at a point are in equilibrium, when their resultant is ————————

(a) zero (b) least

(c) greatest (d) equal

3. If two forces P and Q have resultant $P \sim Q$, then they are ———— forces.

(a) like (b) unlike

(c) like parallel (d) unlike parallel

4. If the line of action of a force passes through the point O, then the moment of that force about that point is ————————.

(a) positive (b) negative

(c) zero (d) not defined

5. If three coplanar forces are in equilibrium, then they are ———— forces.

(a) concurrent (b) parallel

(c) (a) and (b) (d) (a) or (b)

6. Let P, Q and R be coplanar forces acting on a rigid body. If it is in equilibrium, which of the following cannot be true?

(a) P and Q - like parallel

(b) P and Q - unlike parallel

(c) P and Q - couple forces

(d) P, Q, R - like parallel

7. The coefficient of friction $\mu =$ ————————

(a) FR (b) $\frac{R}{F}$

(c) $\frac{F}{R}$ (d) $\tan^{-1} \lambda$

8. When one body in contact with another is in equilibrium, the friction exerted is called ————————

(a) limiting friction (b) statical friction

(c) dynamical friction (d) passive friction

9. The Cartesian equation of the catenary is ————————

(a) $y = c \sin h \frac{x}{c}$ (b) $y = c \cos \frac{x}{c}$

(c) $y = c \sin \frac{x}{c}$ (d) $y = c \cos h \frac{x}{c}$

10. Which of the following is true in a catenary?

(a) $s^2 = y^2 + c^2$

(b) $y^2 = c^2 + s^2$

(c) $y^2 = s^2 - c^2$

(d) $y^2 = c^2 - s^2$

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 250 words.

11. (a) State and prove Lami's theorem.

Or

(b) The resultant of two forces P, Q acting at a certain angle in X and that of P, R acting at the same angle is also X. The resultant of Q, R again acting at the same angle is Y. Prove that $P = \frac{QR(Q+R)}{Q^2 + R^2 - Y^2}$. Also prove that if $P + Q + R = 0$, $Y = X$.

12. (a) Two men carry a load of 224 kg wt, which hangs from a light pole of length 8m each end of which rests on a shoulder of one of the men. The point from which the load is hung is 2 m nearer to one man than the other. What is the pressure on each shoulder?

Or

(b) Find the resultant of two unlike and unequal parallel forces acting on a rigid body.

13. (a) A uniform beam of length l and weight W hangs from a fixed point by two strings of lengths a and b. Prove that the inclination of the rod to the horizon is $\sin^{-1}\left(\frac{a^2 - b^2}{l\sqrt{2(a^2 + b^2) - l^2}}\right)$.

Or

(b) The altitude of a right cone is h and the radius of its base is a. A string is fastened to the vertex and to a point on the circumference of the circular base and is then put over a smooth peg. The cone rests with its axis horizontal. Show that the length of the string is $\sqrt{h^2 + 4a^2}$.

14. (a) A particle of weight 30 kgs resting on a rough horizontal plane is just on the point of motion when acted on by horizontal forces of 6 kg wt and 8 kg wt at right angles to each other. Find the coefficient of friction between the particle and the plane and the direction in which the friction acts.

Or

(b) A uniform ladder is in equilibrium with on end resting on the ground and the other against a vertical wall, if the ground and wall be both rough, the coefficients of friction being μ and μ^1 respectively and if the ladder be on the point of slipping at both ends, show that θ the inclination of the ladder to the horizon is given by $\tan\theta = \frac{1-\mu\,\mu^1}{2\mu}$.

15. (a) The span of a suspension bridge is 100 meters and the sag at the middle of the chain is 10 metres. If the total load on each chain is 750 quintals, find the greatest tension in each chain and tension at the lowest point

Or

(b) Show that the length of an endless chain which will hang over a circular pulley of radius "a" so as to be in contact with two-thirds of the circumference of the pulley is $a\left[\frac{3}{\log\left(2+\sqrt{3}\right)}+\frac{4\pi}{3}\right]$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 600 words.

16. (a) ABC is a given triangle. Forces P, Q, R acting along the lines OA, OB, OC are in equilibrium. Prove that

(i) $P:Q:R = a^2\left(b^2+c^2-a^2\right):$ $b^2\left(c^2+a^2-b^2\right):c^2\left(a^2+b^2-c^2\right)$ if O is the circum centre of the triangle.

(ii) $P:Q:R = \cos\frac{A}{2}:\cos\frac{B}{2}:\cos\frac{C}{2}$, if O is the in centre of the triangle.

(iii) $P:Q:R = a:b:c$ if O is the orthocentre of the triangle

(iv) $P:QR = OA:OB:OC$ if O is the centroid of the triangle.

Or

(b) E is the middle point of the side CD of a square $ABCD$. Forces 16, 20, $4\sqrt{5}$, $12\sqrt{2}$ kg wt act along AB, AD, EA, CA in the directions indicated by the order of the letters. Show hat they are in equilbrium.

17. (a) State and prove Varigon's theorem on moments.

Or

(b) Forces P, Q, R act along the sides BC, AC, BA respectively of an equilateral triangle. If their resultant is a force parallel to BC through the centroid of the triangle, prove that $Q = R = \frac{1}{2} P$.

18. (a) A uniform rod of length $2l$ rests with its lower end in contact with a smooth vertical wall. It is supported by a string of length a, one end of which is fastened to a point in the wall and the other end to a point in the rod at a distance b from its lower end. If the inclination of the string to the vertical be θ, show that $\cos^2\theta = \frac{b^2\left(a^2 - b^2\right)}{a^2 l\left(2b - l\right)}$.

Or

(b) An isosceles triangular lamina, with its plane vertical rests vertex downwards, between two smooth pegs in the same horizontal line, show that there will be equilbrium if the base makes an angle $\sin^{-1}\left(\cos^2 x\right)$ with the vertical, 2α -being the vertical angle of the lamina and the length of the base being three times the distance between the pegs.

19. (a) Find the conditions for equilibrium of a body on a rough inclined plane under a force parallel to the plane.

Or

(b) A glass rod is balanced partly in and partly out of a cylindrical tumbler with its lower and resting against the vertical side of the tumbler. If α and β be the greatest and least angles which the rod can make with the vertical, show that the angle of friction λ is $\frac{1}{2}\tan^{-1}\left(\frac{\sin^3\alpha - \sin^3\beta}{\sin^2\alpha\cos\alpha + \sin^2\beta\cos\beta}\right)$