(7 pages) **Reg. No. :** .....

**Code No. : 8533** **Sub. Code : HPHM 21**

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2013.

Second Semester

Physics

MATHEMATICAL PHYSICS — II

(For those who joined in July 2012 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL the questions.

Choose the correct answer.

1. A square matrix $A$ possesses an inverse if

   (a) $A$ is real (b) $A$ is imaginary

   (c) $A$ is singular (d) $A$ is non-singular

2. A square matrix is said to be orthogonal if

   (a) $A$ is singular (b) $A$ is non-singular

   (c) $A^T A = 1$ (d) $A = -A^T$

3. Which of the following function of complex variable $z = x + iy$ is analytic?

   (a) $|z|$ (b) $\mathrm{Re}\, z$

   (c) $\log z$ (d) $z^{-1}$

4. A function that is analytic at all points of the $z$-plane and finite at infinity

   (a) must have a singularity

   (b) must be zero

   (c) must be a constant

   (d) cannot exist

5. $J_0(x)$ and $J_1(x)$ are Bessel's function, then $J_1'(x)$ is

   (a) $J_0(x) = \frac{1}{x} J_1(x)$ (b) $J_0(x) + \frac{1}{x} J_1(x)$

   (c) $-J_0(x)$ (d) $J_0(x) - \frac{1}{x^2} J_1(x)$

6. Which of the following is not equal to 1?

   (a) $P_0(x)$ (b) $J_0(x)$

   (c) $H_0(x)$ (d) $L_0(x)$

7. Laplace transform of $t^n e^{at}$ is

(a) $\frac{1}{(s+a)^n}$ (b) $\frac{1}{(s+a)^{n+1}}$

(c) $\frac{n!}{(s+a)^n}$ (d) $\frac{n!}{(s+a)^{n+1}}$

8. Fourier transfer of which of the following functions does not exist?

(a) $e^{-|x|}$ (b) $xe^{-x^2}$

(c) $e^{x^2}$ (d) $e^{-x^2}$

9. The integral formula

$$\int_{x_0}^{x_0+nh} y\,dx = \frac{3}{8}h(y_0+y_n)+3(y_1+y_2+y_4+y_5\cdots+y_{n-1}) + 2(y_3+y_6+\cdots+y_{n-3})$$

is called

(a) Simpson 3/8 rule (b) Simpson 1/3 rule

(c) Trapezoidal rule (d) Weddle's rule

10. Runge-Kutta formula for solving the differential equation involves an error of

(a) second order (b) third order

(c) fourth order (d) fifth order



PART B — (5 × 5 = 25 marks)

Answer ALL the questions, choosing either (a) or (b).

11. (a) (i) Define symmetric and anti-symmetric matrix.

(ii) Show that the diagonal elements of a Skew-Hermitian matrix are either zero or pure imaginary numbers.

Or

(b) State and prove Cayley-Hamilton theorem.

12. (a) Find the most general analytic function $f(z)=u+iv$ for which $v=xy$.

Or

(b) If $AB$ is an arc of a circle $|z|=R$ having $\theta_1 \le \theta \le \theta_2$ and $\lim_{R\to\infty} z f(z)$ tends uniformly to $b$; then $\lim_{R\to\infty} \int_{BA} f(z)\,dz = ib(\theta_2-\theta_1)$.

13. (a) Write Rodrigue's formula for Laguerre polynomials. Hence find $L_0(x)$, $L_1(x)$ and $L_2(x)$.

Or

(b) Using recurrent relation prove that $\frac{d}{dx}[x^{-n}J_n(x)] = -x^{-n}J_{n+1}(x)$.

14. (a) Find the sine transform of $\frac{e^{-ax}}{x}$.

Or

(b) Find the inverse Laplace transform of $\frac{e^{-\pi s}}{s^2+1}$.

15. (a) Apply Newton-Raphson method to solve the following system of equations $x^2+2y^2=3.1$, $xy=7.1$.

Or

(b) Use Simpson's one-third rule to evaluate the approximation value of $\int_0^1 \frac{dx}{1+x}$ correct to 3 decimals taking $h=0.25$.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL the questions, choosing either (a) or (b).

16. (a) Find the eigen values and normalized eigen vectors of the matrix $A=\begin{bmatrix}3 & 1 & 4\\ 0 & 2 & 6\\ 0 & 0 & 5\end{bmatrix}$.

Or



(b) Find the transpose and inverse of the matrix $A=\begin{bmatrix}-1 & 5 & -7\\ -6 & 4 & 0\\ 0 & 3 & 1\end{bmatrix}$.

17. (a) Evaluate the integral $\int_0^{\pi} \frac{a\,d\varphi}{a^2\cos^2\varphi}$.

Or

(b) State and prove Taylor theorem for a complex function.

18. (a) State and prove generating function for $J_n(x)$.

Or

(b) Prove that

$(n+1)L_{n+1}(x)=(2n+1-x)L_n(x)-nL_{n-1}(x)$.

19. (a) Obtain the derivatives of the Fourier sine and cosine transforms.

Or

(b) Find the inverse Laplace transform of $\frac{e^{-1/s}}{s}$.

20. (a) Using Newton-Raphson method, find $\sqrt{8}$.

Or

(b) Given that $\frac{dy}{dx} = \log(x+y)$ with the initial condition that $y=1$ when $x=0$. Use Euler's modified method to find $y$ for $x=0.2$ and $x=0.5$.

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