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Reg. No. : .....

Code No. : 20301 E **Sub. Code : SMMA 54**

B.Sc. (CBCS) DEGREE EXAMINATION,
NOVEMBER 2021.

Fifth Semester

Mathematics — Core

TRANSFORMS AND THEIR APPLICATIONS

(For those who joined in July 2017-2019)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer.

1. $F\left[e^{iax} f(x)\right]$ = _____.

   (a) $F(s+a)$ (b) $F(s-a)$

   (c) $-F(s+a)$ (d) $-F(s-a)$

2. $F[f(x)\cos ax]$ = _____.

   (a) $2[f(s+a)+f(s-a)]$ (b) $\frac{1}{2}[f(s+a)-f(s-a)]$

   (c) $\frac{1}{2}[f(s+a)+f(s-a)]$ (d) $2[f(s+a)-f(s-a)]$

3. $F_s\left[\frac{1}{x}\right]$ = _____.

(a) $\frac{\pi}{2}$ (b) 2π

(c) $\sqrt{\frac{2}{\pi}}$ (d) $\sqrt{\frac{\pi}{2}}$

4. $\int_0^\infty e^{-x^2} dx$ = _____.

(a) $\frac{\sqrt{\pi}}{2}$ (b) $\sqrt{\frac{\pi}{2}}$

(c) $\sqrt{\pi}$ (d) $\sqrt{\frac{2}{\pi}}$

5. $F_s\left[e^{ax}\right]$ in $(0, l)$ is _____.

(a) $\int_0^l e^{ax} \sin\frac{n\pi x}{l} dx$ (b) $\int_0^l e^{ax} \frac{\sin n\pi x}{l} dx$

(c) $\int_0^l e^{ax} \sin n\pi x \, dx$ (d) $2\int_{-l}^{l} \sin n\pi x \, dx$

6. $F_c[x]$ in $(0, \pi)$ is _____.

(a) $\frac{1}{n^2}\left[(-1)^n + 1\right]$ (b) $\frac{(-1)^n}{n^2}$

(c) $\frac{1}{n^2}\left[1 - (-1)^n\right]$ (d) $\frac{1}{n^2}\left[(-1)^n - 1\right]$

7. $Z(n)$= _____.

(a) $\frac{z}{(z-1)^2}$ (b) $\frac{z}{(z+1)^2}$

(c) $\frac{z^2}{(z-1)^2}$ (d) $\frac{z^2}{(z+1)^2}$

8. $Z\left[\sin\frac{n\pi}{2}\right]$ = _____.

(a) $\frac{z}{z^2+1}$ (b) $\frac{z^2}{z^2+1}$

(c) $\frac{z^2}{(z+1)^2}$ (d) $\frac{z^2}{z^2-1}$

9. $Z^{-1}\left[\frac{z}{z-a}\right]$ = _____.

(a) a (b) na

(c) a^n (d) $\frac{1}{a^n}$

10. $Z^{-1}\left[e^{\frac{1}{z}}\right]$ = _____.

(a) $\frac{1}{n}$ (b) $n!$

(c) $\frac{1}{n!}$ (d) n

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

Answer should not exceed 250 words.

11. (a) Show that $F\left[\frac{d^n + (x)}{dx^n}\right] = (-is)^n F(s)$, where $F(s) = F[f(x)]$.

Or

(b) State and prove the Parseval's identity for Fourier Transform.

12. (a) Find $F_c\left[e^{-x^2}\right]$.

Or

(b) Find $F_s\left[\frac{e^{-a^x}}{x}\right]$.

13. (a) Find the finite Fourier Cosine transform of $f(x) = \left(1 - \frac{x}{\pi}\right)^2$.

Or

(b) Find the finite Fourier Sine transform of $f(x) = x^3$ in $(0, 4)$.

14. (a) Prove that $Z\left(\frac{1}{n+1}\right) = z \log\left(\frac{z}{z-1}\right)$.

Or

(b) Find $Z\left[n^2\right]$.

15. (a) Find $Z^{-1}\left[\frac{z-4}{(z+2)(z+3)}\right]$.

Or

(b) Find $Z^{-1}\left[\frac{z^3}{(z-1)^2(z-2)}\right]$.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Find the Fourier Transform of $f(x)=\begin{cases}1-x^2, & \text{in } |x|\le 1\\ 0, & \text{in } |x|>1\end{cases}$. Hence find $\int_0^\infty \frac{\sin s - s\cos s}{s^3}\cos\frac{s}{2}ds$.

Or

(b) Prove that $f(x)=e^{-\frac{x^2}{2}}$ is self reciprocal under Fourier transform.

17. (a) If $f(x)=\frac{1}{x\left(a^2+x^2\right)}$, then find the Fourier Sine transform of $f(x)$.

Or

(b) Find the Fourier cosine transform of (i) $f(x)=e^{-ax}\cos ax$ and (ii) $f(x)=e^{-ax}\sin ax$.

18. (a) Find the finite Fourier sine and cosine transform of $f(x) = e^{ax}$ in $(0, l)$.

Or

(b) Find the finite Fourier Sine transform of $f(x) = \cos kx$ in $0 < x < \pi$.

19. (a) (i) Find $Z\left(\sin\frac{n\pi}{2}\right)$ and $Z\left(\cos\frac{n\pi}{2}\right)$.

(ii) State and prove the initial value theorem for ***Z***-transform.

Or

(b) (i) Find $Z\left[t^2 e^{-t}\right]$.

(ii) Prove that $Z[nf(n)] = -z\frac{d}{dz}\{F(z)\}$, where $F(z) = Z[f(n)]$.

20. (a) Find $Z^{-1}\left[\frac{z^2 + 2z}{z^2 + 2z + 4}\right]$.

Or

(b) Find the inverse Z-transform of $\frac{z}{(z-1)(z^2+1)}$ using residue theorem.

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