(6 pages) **Reg. No. :** ....

Code No. : 30580 E **Sub. Code : SMMA 63**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2020.

Sixth Semester

Mathematics — Core

GRAPH THEORY

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer.

1. The number of edges in $K_{3,4}$ is

(a) 7 (b) 12

(c) 3 (d) 4

2. If $\delta = 8$ for a regular graph then $\Delta =$ ————.

(a) 8 (b) 7

(c) 9 (d) 16

3. If e is a bridge of graph G then $w(G-e)=$ ________.

(a) $w(G)-1$ (b) $w(G)$

(c) $w(G)+1$ (d) $2w(G)$

4. Number of cut points of C_4 is

(a) 1 (b) 0

(c) 3 (d) 2

5. With usual notations, $|V|-|E|+|F|=$ ________.

(a) 2 (b) 1

(c) 0 (d) 3

6. If T is a (p, q) tree then which statement is false?

(a) T is a connected acyclic graph

(b) T is a connected regular graph

(c) T is a connected graph, $q=p-1$

(d) T is an acyclic graph, $q=p-1$

7. Which of the following is a planar graph?

(a) K_7 (b) K_6

(c) K_5 (d) K_4

8. Chromatic number of $\overline{K}_5$ is ————.

(a) 2 (b) 3

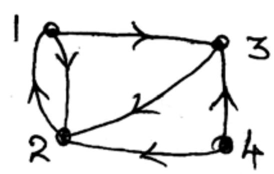
(c) 5 (d) 1

9. If $f(G,\lambda) = \lambda^5 - 7\lambda^4 + 19\lambda^3 - 23\lambda^2 + 10\lambda$ then the number of points in G is

(a) 4 (b) 5

(c) 7 (d) 10

10. What is the in-degree of 2 in the following diagraph?



(a) 1 (b) 2

(c) 3 (d) 4

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that any self complementary graph has $4n$ or $4n+1$ points.

Or

(b) Let G be a $k-$regular bipartite graph with bipartition (V_1, V_2) and $k>0$. Prove that $|V_1| = |V_2|$.

12. (a) Is $P = (4, 4, 4, 2, 2, 2)$ a graphic sequence? If yes then draw a graph for p.

Or

(b) In a graph G, if $\delta \geq k$ then show that G has a path of length k.

13. (a) Prove that every tree has a centre consisting of either one point or two adjacent points.

Or

(b) Prove that $C(G)$ is well-defined.

14. (a) Prove that K_5 is non-planar.

Or

(b) Show that every uniquely $n-$colourable graph is $(n-1)$ connected.

15. (a) If two digraphs are isomorphic then prove that corresponding points have the same degree pair.

Or

(b) Prove that

$f(k_n, \lambda) = \lambda(\lambda-1)(\lambda-2)....(\lambda-n+1)$.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) If G is a (p, q) graph without triangles then prove that $q \leq \left[\frac{p^2}{4}\right]$.

Or

(b) (i) Show that every graph is an intersection graph.

(ii) Let G be a (p, q) graph. Prove that $L(G)$ is a (q, q_L) graph where $q_L = \frac{1}{2}\left(\sum_{i=1}^{p} d_i^2\right) - q$.

17. (a) Show that a graph G with atleast two points is bipartite iff all its cycles are of even length.

Or

(b) (i) Define vertex connectivity and edge connectivity of a graph.

(ii) With usual notations, prove that $k \leq \lambda \leq \delta$.

18. (a) Prove that a connected graph G is Eulerian iff every point of G has even degree.

Or

(b) State and prove Dirac's theorem.

19. (a) Prove that a graph can be embedded in the surface of a sphere iff it can be embedded in a plane.

Or

(b) Show that

$$\chi'(k_n) = \begin{cases} n & \text{if } n \text{ is odd} (n \neq 1) \\ n-1 & \text{if } n \text{ is even} \end{cases}$$

20. (a) Prove that a graph G with $n \geq 2$ points is a tree iff $f(G, \lambda) = \lambda(\lambda-1)^{n-1}$.

Or

(b) Prove that a weak digraph D is Eulerian iff every point of D has equal in-degree and out-degree.

———————