(8 pages) **Reg. No. :** ....

Code No. : 41164 E **Sub. Code : JAMA 21/ SAMA 21**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2019.

Second Semester

Mathematics – Allied

VECTOR CALCULUS AND FOURIER SERIES

(For those who joined in July 2016 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. If $\vec{r} = x\vec{i} + y\vec{j} + 2\vec{k}$ then $\nabla \times \vec{r}$ = ————

   (a) 0 (b) $\vec{0}$

   (c) 1 (d) 3

2. If $\nabla \phi$ is solenoid then $\nabla^2\phi$ = ————

   (a) 0 (b) $\vec{0}$

   (c) 1 (d) 3

3. $\nabla 2\phi$ = ————

   (a) $\nabla\nabla\varphi$ (b) $\nabla \times \nabla\varphi$

   (c) $\nabla . \nabla\phi$ (d) $\nabla\phi^2$

4. $div\, curl\, \vec{f}$ = ————

   (a) 0 (b) $\vec{0}$

   (c) $\vec{f}$ (d) $2\vec{f}$

5. The line integral of $\vec{f}$ over C is denoted by ————

   (a) $\int_C \vec{f}\, d\vec{r}$ (b) $\int_C \vec{f} \times d\vec{r}$

   (c) $\int_C \vec{f} . d\vec{r}$ (d) None of these

6. Stokes theorem connects ————

   (a) line and double integrals

   (b) line and surface integrals

   (c) surface and volume integrals

   (d) line and volume integrals

7. $\int_0^1 \int_0^1 xy^2 \, dy \, dx$ = ________

(a) 0 (b) 1

(c) $\frac{4}{3}$ (d) $\frac{1}{3}$

8. If $x + y = v$, $2x - 3 = u$ then $\frac{\partial(v,u)}{\partial(x,y)}$ = ________

(a) $-\frac{1}{5}$ (b) $\frac{1}{5}$

(c) 5 (d) -5

9. $f(x) = x \sin x$ is ________

(a) an even function (b) an odd function

(c) neither (a) nor (b) (d) a constant function

10. If $f(x)$ is an even function then $\int_{-a}^{a} f(x)\,dx$ = ________

(a) 0 (b) $2\int_0^a f(x)\,dx$

(c) $2\int_{-a}^{0} f(x)\,dx$ (d) Both (b) and (c)

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 250 words.

11. (a) If $\bar{r} = \bar{a} \cos wt + \bar{b} \sin wt$, where $\bar{a}$ and $\bar{b}$ are constant vector, prove that $\bar{r} \times \frac{d\bar{r}}{dt} = w(\bar{a} \times \bar{b})$ and $\frac{d^2\bar{\partial}}{dt^2} + w^2\bar{r} = \bar{0}$.

Or

(b) Find the unit normal to the surface $x^3 - xyz + z^3 = 1$ are $(1,1,1)$.

12. (a) Evaluate $I = \int_0^a dx \int_0^{b\sqrt{1-(x^2/a^2)}} x^3 y \, dy$.

Or

(b) Evaluate $I = \int_0^{\pi} \int_0^{\pi/2} \int_0^{k} r^2 \sin\theta \, dr \, d\theta \, d\phi$.

13. (a) Evaluate $\int_{(1,1)}^{(4,2)} \bar{f}.\,d\bar{r}$ along the parabola $y^2 = x$ then prove that $\bar{f} = (x+y)\bar{i} + (y-x)\bar{j}$.

Or

(b) Find $t = 1$ the work done by the force $\bar{F} = 3xy\,\bar{i} + 52\,\bar{j} + 10x\,\bar{k}$ along the curve $C : x = t^2 + 1, y = 2t^2, z = t^3$.

14. (a) State Green's and stokes theorem.

Or

(b) By using Stoke's theorem prove that $\int_C \bar{r}.\,d\bar{r} = 0$ where $\bar{r} = x\bar{i} + y\bar{r} + 2\bar{k}$.

15. (a) Find the Fourier series for the function $f(x) = \begin{cases} -1 \text{ in } -\pi \le x < 0 \\ 1, \text{ in } 0 \le x \le \pi \end{cases}$.

Or

(b) Find the sine series of $f(x) = k$ in $0 < x < \pi$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 600 words.

16. (a) Find the value of a so that the vector function $\bar{f} = (axy - z^3)\bar{i} + (a-z)\bar{j} + (1-a)$ $xz^2\bar{x}$ is irrigational. Hence find $\varphi(x, y, z)$ such that $\bar{f} = \nabla\phi$.

Or

(b) Prove:

(i) $\nabla.(\bar{f} \times \bar{g}) = g.(\nabla \times f) - \bar{f}.(\nabla \times \bar{f})$.

(ii) $\nabla \times (\nabla \times \bar{f}) = \nabla(\nabla.\bar{f}) - \nabla^2 \bar{f}$.

17. (a) Evaluate $I = \iiint_D xyz\, dx\, dy\, dz$ where D is the region bounded by the positive ocant of the sphere $x^2 + y^2 + z^2 = a^2$.

Or

(b) Evaluate $I = \iiint_D \frac{dx\, dy\, dz}{(x+y+z+1)^3}$ where D is the region bounded by the region $x = 0, y = 0, z = 0$ $x + y + z = 1$.

18. (a) Evaluate $\int_C \vec{f}.d\vec{r}$ where $\vec{f}=(x^2+y^2)\vec{i}-2xy\,\vec{j}$ and the curve C is the rectangle in the $x-t$ plane bounded by $y=0, y=b, x=0, x=a$.

Or

(b) Evaluate $\iint_C \vec{f}.\vec{n}\,ds$ where $\vec{f}=(x+y^2)\vec{i}-2x\vec{j}+2yz\vec{k}$ and S is the surface of the plane $2x+y+yz=6$ is the octant.

19. (a) Verify Stoke's theorem for $\vec{f}=y^2\vec{i}+y\vec{z}-xz\,\vec{k}$ and S is the suffer half of the sphere $x^2+y^2+z^2=a^2, z\geq 0$.

Or

(b) Verify Green's theorem for $\vec{f}=(x^3-xy^3)\vec{i}+(y^2-2xz)dy$ where C is the square with vertices $(0,0),(2,0),(2,2)$ and $(0,2)$.

20. (a) Find the Fourier series for the functions $f(x)=x^2$ in $-\pi\leq x\leq\pi$ and hence deduce that $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+....=\frac{\pi^2}{6}$.

Or

(b) Expand $y=\cos 2x$ in a series of sines in the interval $(0,\pi)$.
