Reg. No. : ....

Code No. : 5762 **Sub. Code : WMAM 23**

M.Sc. (CBCS) DEGREE EXAMINATION,
APRIL 2024.

Second Semester

Mathematics — Core

PARTIAL DIFFERENTIAL EQUATIONS

(For those who joined in July 2023 onwards)

Time : Three hours Maximum : 75 marks

PART A — (15 × 1 = 15 marks)

Answer ALL the questions.

Choose the correct answer :

1. The Poisson equation is ___________.

(a) $\nabla^2 u = f(x, y, z)$

(b) $u_{t_t} + c^2\nabla^4 u = 0$

(c) $\nabla^4 u = \nabla^2(\nabla^2 u) = 0$

(d) $\nabla^2 u + \lambda u = 0$

24. (a) State and prove mean value theorem.

Or

(b) Solve the Dirichilet problem.

$$\nabla^2 u = -2y,\ 0 < x < 1,\ 0 < y < 1$$

$$u(0, y) = 0,\ u(1, y) = 0,\ 0 \le y \le 1$$

$$u(x, 0) = 0,\ u(x, 1) = 0,\ 0 \le x \le 1$$

25. (a) Solve the boundary value problem

$$\frac{1}{r}\frac{\partial G}{\partial r}\left(\frac{r\partial u}{\partial r}\right) + \frac{\partial^2 u}{\partial z^2} + k^2 u = 0,\ r \ge 0,\ z > 0,$$

$$\frac{\partial u}{\partial z} = \begin{cases} 0, r > a, z = 0 \\ c, r < a, z = 0, c = \text{constant.} \end{cases}$$

Or

(b) Prove that Greens function is symmetric.

2. If the canonical form of the equation is ___________, then the general solution can be immediately ascertained.
(a) General (b) Simple
(c) Original Solution (d) Zero

3. If $B^2 - 4AC > 0$, the equation is of ___________ type.
(a) Parabolic (b) Elliptic
(c) Hyperbolic (d) None of the above

4. Determine the solution of initial value problem
$u_{tt} - c^2 u_{xx} = x$, $u(x, 0)$, $u_t(x, 0) = 3$.
(a) $u(x, t) = x + \left(\frac{1}{c}\right) \sin x \sin ct$
(b) $u(x, t) = 3t + \frac{1}{2} x t^2$
(c) $u(x, t) = t$
(d) $u((x, t) = \cos x \cos ct + t)$

5. Determine the solution of initial value problem
$u_{tt} - c^2 u_{xx} = x$, $u(x, 0) = x^3$, $u_t(x, 0) = x$
(a) $u\left((x, t) = \cos x \cos ct + \left(\frac{t}{c}\right)\right)$
(b) $u\left((x, t) = \cos ct + \left(\frac{t}{c}\right)\right)$
(c) $u(x, t) = x^3 + 3c^2 x t^2 + xt$
(d) $u(x, t) = \sin x \cos ct + x^2 t + \frac{1}{3} c^2 t^3$

6. The D'Alembert solution has the form ___________.
(a) $u(x, t) = \frac{1}{2}[f(x + ct) + f(x - ct)]$
(b) $u_t - k(u_{x_x} + u_{y_y} + u_{z_z}) = 0$
(c) $u(x, t) = [f(x + ct) + f(x - ct)]$
(d) $u(x, t) = \frac{1}{2}[f(x + ct) f(x - ct)]$

7. The Lapalce transformation of $1/\sqrt{t}$ is
(a) $\frac{1}{s} - a$ (b) $\left(\frac{1}{s - a}\right)$
(c) $s - a$ (d) $\sqrt{\frac{\pi}{8}}$

8. The Laplace transformation of $1/\sqrt{\pi t}(1 + 2at)e^{at}$ is
(a) $\frac{s}{s - a\sqrt{s - a}}$ (b) $\frac{a}{s - a}$
(c) $\frac{1}{\sqrt{s} + a}$ (d) $s - a$

9. Which Boundary condition $\left(\frac{\partial u}{\partial n}\right)$ is prescribed on a boundary?

(a) Neumann condition

(b) Mixed condition

(c) Dirichlet condition

(d) All (a), (b), (c)

10. Schrodinger equation (time dependent) ___________.

(a) $\nabla^2 u + [\lambda - q(x)]u = 0$

(b) $\nabla u + q(x) = 0$

(c) $\nabla^3 u + [\lambda - q(x)]u = 0$

(d) $\nabla^2 u + \lambda t = 0$

11. In free space the Poisson equation become

(a) Maxwell equation

(b) Ampere equation

(c) Lapalce equation

(d) Steady state equation

12. Which one of the following is a property of the solutions to the Laplace equation : $\nabla^2 f = 0$?

(a) The solution have neither maxima nor minima anywhere except at the boundaries

(b) The solution are not separable in the coordinates

(c) The solution are not continuous

(d) The solution are not dependent on the boundary condition

13. The Greens function is ___________.

(a) Antisymmetric (b) Symmetric

(c) Operators (d) Delta

14. Let us consider the differential equation $\frac{d^2y}{dx^2} = f(x),\ y(0) = y(1) = 0$ if $G(x, s)$ be the greens function. Then which of the following is correct?

(a) $G(x, s) = G(s, x)$ for all x and s

(b) $G(x, s) = G(s, x)$ for some x and s

(c) $G(x, s) = -G(s, x)$ for all x and s

(d) $G(x, s) = -G(s, x)$ for some x and s

15. The Dirac delta function is also known as ____________.

(a) Impulse (b) Greens function

(c) Unit Dirac (d) Unit impluse

PART B — (5 × 4 = 20 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Prove that the equation $u_{xx} + x^2 u_{yy} + 0$ is elliptic.

Or

(b) Explain hyperbola type in equation with constant co-efficient.

17. (a) Determine the solution of the initial value problem $u_{tt} - c^2 u_{xx} = 0$, $u(x, 0) = \sin x$, $u_t(x, 0) = x^2$.

Or

(b) Solve the initial value problem $u_{xx} + 2u_{xy} - 3u_{yy} = 0$, $u(x, 0) = \sin x$, $u_y(x, 0) = x$.

18. (a) Solve $u_{tt} = c^2 u_{xx}$, $0 < x < 1$, $t > 0$,
$u(x, 0) = x(1 - x)$, $u_t(x, 0) = 0$, $0 \le x \le 1$,
$u(0, t) = u(1, t) = 0$, $t > 0$.

Or

(b) Obtain the solution $u_t = 4u_{xx}$, $0 < x < 1$, $t > 0$,
$u(x, 0) = x^2(1 - x)$, $0 \le x, \le 1$, $u(0, t) = 0$,
$u(l, t) = 0$, $t \ge 0$.

19. (a) Determine the solution $\nabla^2 u = 0$, $1 < r < 2$, $0 < \theta < \pi$,
$u(1, \theta) = \sin\theta$, $u(2, \theta) = 0$, $0 \le \theta \le \pi$,
$u(r, 0) = 0$, $u(r, \pi) = 0$, $1 \le r \le 2$.

Or

(b) Solve $\nabla^2 u = 0$, $1 < r < 2$, $0 < \theta < 2\pi$,
$u_r(1, \theta) = \sin\theta$, $u_r(2, \theta) = 0$, $0 \le \theta \le 2\pi$.

20. (a) Obtain the solution of Laplace equation $\nabla^2 u = 0$, $0 < r < \infty$, $0 < \theta < 2\pi$,
$u(r, 0+) = u(r, 2\pi -) = 0$.

Or

(b) Prove that $\frac{\partial G}{\partial n}$ is discontinuous at (ξ, η); in particular
$\lim_{\epsilon \to 0} \iint_{C_\epsilon} \frac{\partial G}{\partial n} ds = 1$, $C_\epsilon : (x - \xi)^2 + (y - \eta)^2 = \epsilon^2$.

PART C — (5 × 8 = 40 marks)

Answer ALL the questions, choosing either (a) or (b).

21. (a) Consider the current $I(x, t)$ and the potential $v(x, t)$ at a point x and time t of a uniform electric transmission line with resistance R, inductance L, capacity C and leakage conductance G per unit length. Find the equations for I and V in the following cases :

(i) Lossless transmission line ($R = G = 0$),

(ii) Ideal submarine cable ($L = G = 0$),

(iii) Heaviside's distortion less line $\frac{R}{L} = \frac{G}{C} =$ Constant $= k$.

Or

(b) Classify the equations and reduce into canonical form.

(i) $yu_{xx} - xu_{yy} = 0$, $x > 0$, $y > 0$;

(ii) $u_{xx} - (\sec^4 x)u_{yy} = 0$;

(iii) $u_{xx} + 6u_{xy} + 9u_{yy} + 3yu_y = 0$.

22. (a) Solve $xu_{xx} - x^3u_{yy} - u_x = 0$, $x \neq 0$, $u(x, y) = f(y)$ on $y - \frac{x^2}{2} = 0$ for $0 \le y \le 2$,

$u(x, y) = g(y)$ on $y + \frac{x^2}{2} = 4$ for $2 \le y \le 4$.

Or

(b) Find the solution of the characteristics initial value problem.

$y^3u_{xx} - yu_{yy} + u_y = 0$,

$u(x, y) = f(x)$ on $x + \frac{y^2}{2} = 4$ for $2 \le x \le 4$,

$u(x, y) = g(x)$ on $x - \frac{y^2}{2} = 0$ for $0 \le x \le 2$, with $f(2) = g(2)$.

23. (a) State and prove uniqueness theorem of the vibrating string problem.

Or

(b) Find the temperature distribution in a rod length l, the faces are insulated and the initial temperature distribution is given by $x(l - x)$.