

(7 pages)

Reg. No. :

Code No. : 9360

Sub. Code : HMAM 24

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2015.

Second Semester

Mathematics

ORDINARY DIFFERENTIAL EQUATIONS

(For those who joined in July 2012 onwards)

Time : Three hours

Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. For the homogeneous equation $y'' + P(x)y' + Q(x)y = 0$, where $P(x), Q(x)$ are functions of x alone, _____ for all x is always a solution
- (a) $y(x) = 0$ (b) $y'(x) = 0$
(c) $y''(x) = 0$ (d) None of these

2. If $f(x)$ and $g(x)$ are two functions defined on $[a, b]$ with $f(x)$ identically zero, then $f(x)$ and $g(x)$ are _____
- (a) Linearly independent
(b) Linearly dependent
(c) $f(x) \neq g(x)$
(d) None of these
3. The field of special functions was cultivated by _____
- (a) Gauss (b) Abel
(c) Both (d) None of these
4. The series $\sum_{n=0}^{\infty} x^n$ is _____ series $\forall |x| < 1$
- (a) Convergent (b) Divergent
(c) Oscillating (d) None of these
5. _____ is the regular singular point of the equation $x^2 y'' + pxy' + qy = 0$
- (a) $x = 0$ (b) $x = 1$
(c) $x = -1$ (d) None of these

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6. With usual notation, $F(a, b, c, x)$ _____
 $F(b, a, c, x)$

- (a) equals (b) is not equal to
 (c) data inadequate (d) none of these

7. $P_1(x) =$ _____

- (a) 1 (b) 0
 (c) x (d) x^2

8. For any value of p , the function $J_p(x)$ has a/an _____
 number of positive zeros

- (a) infinite (b) finite
 (c) p (d) none of these

9. $P_n(-1) =$ _____

- (a) 1 (b) -1
 (c) $(-1)^n$ (d) $(-1)^{n-1}$

10. $\Gamma(p+1) =$ _____

- (a) $(p+1)\Gamma(p)$ (b) $\Gamma(p-1)$
 (c) $p\Gamma(p)$ (d) None of these

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PART B — (5 × 5 = 25 marks)

Answer ALL questions choosing either (a) or (b).

11. (a) If y_g is the general solution of equation $y''+P(x)y'+Q(x)y=0$ and y_p is any particular solution of equation $y''+P(x)y'+Q(x)y=R(x)$, then prove that $y_g + y_p$ is the general solution of $y''+P(x)y'+Q(x)y=R(x)$.

Or

- (b) Show that $y = c_1e^x + c_2e^{2x}$ is the general solution of $y''-3y'+2y=0$ on any interval, and find the particular solution for which $y(0) = -1$ and $y'(0) = 1$.

12. (a) Find the differential equation satisfied by the function $y = (1+x)^2$ and then solve this equation by power series.

Or

- (b) Find the power series solution of the form $\sum a_n x^n$ of the differential equation $xy' = y$.

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[P.T.O.]



13. (a) Find the indicial equation and its roots for the differential equation $4x^2y'' + (2x^4 - 5x)y' + (3x^2 + 2)y = 0$.

Or

- (b) Verify that $\tan^{-1} x = xF\left(\frac{1}{2}, 1, \frac{3}{2}, -x^2\right)$ by examining the series expansions of the function.

14. (a) Prove that $\int_{-1}^1 P_m(x) P_n(x) dx = 0$ if $m \neq n$.

Or

- (b) Prove that $\frac{d}{dx} J_0(x) = -J_1(x)$.

15. (a) If the homogeneous system $\frac{dx}{dt} = a_1(t)x + b_1(t)y$, $\frac{dy}{dt} = a_2(t)x + b_2(t)y$ has two solutions $x = x_1(t)$, $y = y_1(t)$ and $x = x_2(t)$, $y = y_2(t)$ on $[a, b]$ then prove that $x = c_1x_1(t) + c_2x_2(t)$, $y = c_1y_1(t) + c_2y_2(t)$ is also a solution on $[a, b]$ for any constant's c_1 and c_2 .

Or

- (b) Find the exact solution of the initial value problem $y' = 2x(1 + y)$, $y(0) = 0$ starting with $y_0(x) = 0$, calculate $y_1(x)$, $y_2(x)$.

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PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).

16. (a) Derive the method of variation of parameters.

Or

- (b) Find a particular solution of the equation $y'' + 4y = \tan 2x$.

17. (a) Consider the equation $y'' + xy' + y = 0$

- (i) Find its general solution $y = \sum a_n x^n$ in the form $y = a_0 y_1(x) + a_1 y_2(x)$ where $y_1(x)$ and $y_2(x)$ are power series.

- (ii) Use the ratio test to verify that the two series $y_1(x)$ and $y_2(x)$ converge for all x .

Or

- (b) Solve the Legendre's equation $(1 - x^2)y'' - 2xy' + p(p+1)y = 0$ where p is constant.

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18. (a) Verify that the origin is a regular singular point of the equation $4xy'' + 2y' + y = 0$ and calculate two independent Frobenius series solutions.

Or

- (b) Find two independent Frobenius series solution of the equation $xy'' + 2y' + xy = 0$.

19. (a) Derive the Bessel function of the first kind of order p .

Or

- (b) Derive Rodrigues' formula.

20. (a) Find the general solution of the system

$$\begin{cases} \frac{dx}{dt} = -4x - y \\ \frac{dy}{dt} = x - 2y \end{cases}$$

Or

- (b) State and prove Picard's theorem.
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