(6 pages) **Reg. No. :** ....................................

**Code No. : 6369** **Sub. Code : HMAM 43**

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2016.

Fourth Semester

Mathematics

ANALYTIC NUMBER THEORY

(For those who joined in July 2012 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. Which one of the following is not correct?

(a) $n \mid n$ (b) $1 \mid n$

(c) $n \mid 0$ (d) $0 \mid 0$.

2. $[a,b]=0$ if

(a) $a=0$ (b) $b=0$

(c) (a) or (b) (d) (a) and (b).

3. The value of $\phi(3)$ is

(a) 1 (b) 2

(c) 3 (d) 0.

4. $\Lambda(4)=$ ——————

(a) 0 (b) $\log 2$

(c) $\log 3$ (d) $\log 1$.

5. $\sigma_\alpha(n)=$ ——————

(a) $\sum_{d \mid n} d^2$ (b) $\sum_{d \mid n} d$

(c) $\prod_{d \mid n} d^2$ (d) $\prod_{d \mid n} d$.

6. If $f$ is multiplicative then

(a) $f(1)=0$ (b) $f(1)=1$

(c) $f(m,m)=f(m)f(n)$ (d) none of these.

7. $\xi(s)=$ ——————

(a) $\sum_{n=1}^{\infty} n^s$ (b) $\sum_{n=-\infty}^{\infty} n^s$

(c) $\sum_{1}^{\infty} \frac{1}{n^s}$ (d) $\sum_{-\infty}^{\infty} \frac{1}{n^s}$.

8. If two integers $a$ and $b$ are chosen at random, the probability that they are relatively prime is ________

(a) $\frac{1}{\pi^2}$ (b) $\frac{6}{\pi^2}$

(c) $\pi^2$ (d) $\frac{\pi^2}{6}$

9. $\psi(x) =$ ________

(a) $\sum_{n\le x}\Lambda(n)$ (b) $\sum_{n\le x}\lambda(n)$

(c) $\sum_{n=1}^{\infty}\Lambda(n)$ (d) $\sum_{n=1}^{\infty}\lambda(n)$.

10. The prime number theorem states that as $n \to \infty$

(a) $\pi(n) \sim \frac{\log(n)}{n}$ (b) $\pi(n) \sim \frac{n}{\log n}$

(c) $\pi(n) \sim n\log n$ (d) $\pi(n) \sim 6\log n$.

PART B — (5 × 5 = 25 marks)

Answer ALL questions choosing either (a) or (b).

11. (a) Prove that there are infinitely many primes.

Or

(b) Prove that $n^4 + 4$ is composite if $n > 1$.



12. (a) If $n \ge 1$ then show that $\sum_{d/n}\phi(d) = n$.

Or

(b) If both $g$ and $f * g$ are multiplicative then prove that $f$ is also multiplicaive.

13. (a) If $f$ is an arithmetical function with $f(1) \ne 0$, show that there is a unique arithmetical function $f^{-1}$ such that $f * f^{-1} = f^{-1} * f = I$.

Or

(b) For all $x \ge 1$, prove that

$$\left|\sum_{n\le x}\frac{\mu(n)}{n}\right| \le 1 \text{ with equality holding if } x < 2.$$

14. (a) Define the derivative of an arithmetical function and prove that $(f * g)' = f' * g + f * g'$.

Or

(b) Define the Bell series. And, obtain the Bell series of the Euler's totient $\phi$.

15. (a) Calculate the highest power of 10 that divides 1000!.

Or

(b) For every integer $n \geq 2$, prove that $\pi(n) > \frac{1}{6}\frac{n}{\log n}$.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions choosing either (a) or (b).

16. (a) State and prove the fundamental theorem of arithmetic.

Or

(b) (i) Prove that if $2^n - 1$ is prime then $n$ is prime.

(ii) Prove that $2^n + 1$ is prime then $n$ is a power of 2.

17. (a) Prove that

(i) $\sum_{d/n} \mu(d) = \mu^2(n)$

(ii) $\prod_{t/n} t = n^{d(n)/2}$.

Or

(b) State and prove the Mobius inversion formula.



18. (a) (i) If $f$ and $g$ are multiplicative, prove that $f * g$ is multiplicative.

(ii) If both $g$ and $f * g$ are multiplicative, then prove that $f$ is multiplicative.

Or

(b) Find all integers $n$ such that

(i) $\phi(n) = \frac{n}{2}$

(ii) $\phi(n) = \phi(2n)$

(iii) $\phi(n) = 12$.

19. (a) State and prove the Euler's summation formula.

Or

(b) Derive the Drichlet's asymptotic formula for the partial sums of the divisor function $d(n)$.

20. (a) State and prove the Abel's identity.

Or

(b) Show that the following relations are logically equivalent :

(i) $\lim_{x \to \infty} \frac{\pi(x)\log x}{x} = 1$

(ii) $\lim_{x \to \infty} \frac{\vartheta(x)}{x} = 1$

(iii) $\lim_{x \to \infty} \frac{\psi(x)}{x} = 1$.