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Reg. No. :

Code No. : 9378

Sub. Code : HPHM 21

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2015.

Second Semester

Physics

MATHEMATICAL PHYSICS — II

(For those who joined in July 2012 onwards)

Time : Three hours

Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer :

1. A square matrix A for which $A^2 = A$ is called
 - (a) nilpotent
 - (b) idempotent
 - (c) null
 - (d) periodic
2. A matrix for which every diagonal element is either zero or a pure imaginary number is called
 - (a) Symmetric
 - (b) Anti-symmetric
 - (c) Hermitian
 - (d) Skew Hermitian

3. Taylor's series is called as Maclaurin series, when
 - (a) $Z_0 = 0$
 - (b) $Z_0 > 0$
 - (c) $Z_0 < 0$
 - (d) $Z_0 = \text{infinity}$
4. An analytic function where only singularities in the finite plane are poles is said to be
 - (a) Entire function
 - (b) Mesomorphic function
 - (c) Isolated singularity
 - (d) Essential singularity
5. The Bessel function of second kind $Y_n(X)$ is obtained by taking the linear combination of $J_n(X)$ and $J_{-n}(x)$ is known as
 - (a) Neuman's function
 - (b) Hankel function
 - (c) Bessel co-efficient
 - (d) Jacobi function
6. $J_0(x) =$
 - (a) $1 - \frac{x}{1^2} + \frac{x^2}{2^2} - \dots$
 - (b) $1 + \frac{x^2}{2^2} + \frac{x^4}{2^2 3^2} + \dots$
 - (c) $1 - \frac{x^2}{2^2} + \frac{x^4}{2^2 4^2} - \dots$
 - (d) $1 + \frac{x^2}{2^2} - \frac{x^4}{4^2} + \dots$



7. The transform of a product of two functions is given by

- (a) Fourier integral
- (b) Convolution integral
- (c) Square of convolution integral
- (d) Self convolution integral

8. The value of $L\{\cos at\}$ is

- (a) $\frac{a}{s^2 + a^2}$
- (b) $\frac{s}{s^2 + a^2}$
- (c) $\frac{1}{s^2 - a^2}$
- (d) $\frac{a}{s^2 - a^2}$

9. From the general quadrature formula if substituting $n = 2$ you get

- (a) Simpsons 1/3 rule
- (b) Simpsons 3/8 rule
- (c) Trapezoidal rule
- (d) Weddle's rule

10. The short comings of Runge-Kutta method is, it lacks in estimating the

- (a) value
- (b) accuracy
- (c) error
- (d) root

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PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 250 words.

11. (a) If H is a Hermitian matrix, what kind of matrix is e^{iH} ?

Or

(b) Find the inverse of the matrix

$$\begin{bmatrix} 1 & -1 & 3 \\ -1 & 1 & 2 \\ 3 & 2 & -1 \end{bmatrix}.$$

12. (a) Show that $e^{\sin z}$ is analytical function of complex variable $z = x + iy$.

Or

(b) Find the residues off $f(z) = \frac{ze^z}{(z-a)^3}$ at $z = a$.

13. (a) Express J_5 in terms of J_1 and J_2 .

Or

(b) State and prove the generating function of Laguerre polynomial.

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14. (a) Find the Fourier transform of $e^{-|t|}$.

Or

- (b) Find the Inverse Laplace transform of $\frac{1}{s^2 - 7s + 12}$ by partial fraction method.

15. (a) Use Taylor's series method to find y for $x = 0.1$, correct to four places of decimal if y satisfies $\frac{dy}{dx} = x - y^2$ with $y_0 = 1$, $x_0 = 0$.

Or

- (b) Using Newton-Raphson method, obtain a root of $\sin x = 1 - x$ to three decimals.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 600 words.

16. (a) Diagonalise the matrix

$$A = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Or

- (b) Find the characteristic equation of the matrix $A = \begin{bmatrix} 4 & 3 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 1 \end{bmatrix}$ hence find A^{-1} .

17. (a) Prove that $\cosh(z + 1/z) = a_0 + \sum_{n=1}^{\infty} a_n (z^n + 1/z^n)$, when $a_n = 1/2\pi \int_0^{2\pi} \cos(n\theta) \cosh(2\cos\theta) d\theta$.

Or

- (b) Evaluate the following real integrals

(i) $\int_{-\infty}^{+\infty} \frac{\cos x}{x^4 + 1} dx$

(ii) $\int_{-\infty}^{+\infty} \frac{\cos 4x}{(x^2 + 1)(x^2 + 4)} dx.$

18. (a) Show that when n is integer

$$J_n(x) = \frac{1}{x} \int_0^{\pi} \cos(n\theta - x \sin \theta) d\theta.$$

Or

- (b) State and prove any two recurrence relations of Laguerre's polynomial.



19. (a) Using Laplace transformation solve the following differential equation :

$$\frac{d^2x}{dt^2} + 9x = \cos 2t, \text{ if } x(0) = 1, x(\pi/2) = -1.$$

Or

- (b) Find the Fourier transform of Gaussian distribution function $f(x) = Ne^{-ax^2}$ where N and α are constant.

20. (a) Find the numerical solution of $\frac{dy}{dx} = x + y$ from $x = 0$ to 0.2 by modified Euler's method.

Or

- (b) Using Runge - Kutta method of fourth order solve $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$ with $y(0) = 1$ at $x = 0.2, 0.4$.

