

(8 pages)

Reg. No. :

Code No. : 6353

Sub. Code : HMAM 13

M.Sc. (CBCS) DEGREE EXAMINATION,
NOVEMBER 2013.

First Semester

Mathematics

PROBABILITY AND STATISTICS

(For those who joined in July 2012 onwards)

Time : Three hours

Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. If $\mu = 0$ and $\sigma^2 = 1$, $\Pr\left(|X| \geq \frac{3}{2}\right)$ has the upper bound

- (a) 0 (b) $\frac{3}{2}$
(c) $\frac{4}{9}$ (d) $\frac{9}{4}$

2. If $P(C_1 \cap C_2) = \frac{15}{56}$ and $P(C_1) = \frac{3}{8}$ then value of $P(C_2 / C_1)$ is _____

- (a) $\frac{5}{7}$ (b) $\frac{15}{8}$
(c) $\frac{3}{56}$ (d) $\frac{45}{448}$

3. The variance of binomial distribution $b(n, p)$ is

- (a) np (b) $np(1-p)$
(c) $\sqrt{np(1-p)}$ (d) $p(1-p)$

4. If the random variable X has a poisson distribution such that $\Pr(X=1) = \Pr(X=2)$, then the value of mean is

- (a) 1 (b) 2
(c) 4 (d) 0

5. Let X have a p.d.f $f(x) = \frac{1}{3}$, $x = 1, 2, 3$, zero elsewhere. The p.d.f of $Y = 2X + 1$ is given by $g(y) = \underline{\hspace{2cm}}$, $y = 3, 5, 7$ zero elsewhere

- (a) $\frac{1}{3}$ (b) 3
(c) $\frac{2}{3}$ (d) 1

Page 2

Code No. : 6353



6. $g(y) = \frac{1}{2} e^{-|y|}$, $-\infty < y < \infty$ is called the p.d.f of _____ distribution

- (a) exponential (b) poisson
(c) geometric (d) double exponential

7. If $X_1, X_2, \dots, X_n$ are the random samples with MGF $m(t)$, then the MGF of $Y = \sum_{i=1}^n X_i$ is _____

- (a) $\sum_{i=1}^n m(t)$ (b) $m(t)$
(c) $m\left(\frac{1}{t}\right)$ (d) $[m(t)]^n$

8. Let $X_1, X_2, \dots, X_n$ denote a random sample of size $n \geq 2$ from a distribution that is $n(\mu, \sigma^2)$. The distribution of $\bar{X} = \frac{\sum X_i}{n}$ is

- (a) $n(\mu, \sigma^2)$ (b) $n\left(\mu, \frac{\sigma^2}{n}\right)$
(c) $n\left(\frac{\mu}{n}, \sigma^2\right)$ (d) $n\left(n\mu, \frac{\sigma^2}{n}\right)$

9. A distribution of the discrete type which has a probability of 1 at a single point has been called a _____ distribution

- (a) Limiting (b) Degenerate
(c) Normal (d) Uniform

10. Let Y_n be a random variable whose distribution depends upon the positive integer n . Let C denote a constant which does not depend on n . If $\lim_{n \rightarrow \infty} \Pr(Y_n - c < \varepsilon) = 1$ for every $\varepsilon > 0$, then the random variable converges in probability to

- (a) 0 (b) 1
(c) ∞ (d) c

PART B — (5 × 5 = 25 marks)

Answer ALL questions choosing either (a) or (b).

11. (a) Prove that

$$\frac{P(C_1 \cap C_2 \cap C_3 \cap C_4)}{P(C_3 | C_1 \cap C_2) P(C_4 | C_1 \cap C_2 \cap C_3)} = \frac{P(C_1)P(C_2 | C_1)}{P(C_3 | C_1 \cap C_2)}$$

Or

(b) Let X be a random variable such that $\Pr(X \leq 0) = 0$ and let $\mu = E(X)$ exist. Show that $\Pr(X \geq 2\mu) \leq \frac{1}{2}$.



12. (a) If the m.g.f of a random variable X is $\left(\frac{1}{3} + \frac{2}{3}e^t\right)^5$, find $\Pr(X = 2 \text{ or } 3)$.

Or

- (b) If $(1 - 2t)^{-6}$, $t < \frac{1}{2}$, is the m.g.f of the random variable X , find $\Pr(X < 5.23)$.
13. (a) Find the probability that exactly four observations of a random sample of size 5 from the distribution having p.d.f $f(x) = \frac{x+1}{2}$, $-1 < x < 1$, zero elsewhere, exceed zero.

Or

- (b) Let X have a p.d.f $f(x) = \left(\frac{1}{2}\right)^x$, $x = 1, 2, 3, \dots$ zero elsewhere. Find the p.d.f of $Y = X^3$.
14. (a) Let X be $N(0,1)$. Use m.g.f technique to show that $Y = X^2$ is $\chi^2(1)$.

Or

- (b) Let $\bar{X}$ be the mean of a random sample of size 5 from a normal distribution with $\mu = 0$ and $\sigma^2 = 125$. Determine c so that $\Pr(\bar{X} < c) = 0.90$.

Page 5

Code No. : 6353

15. (a) Let $\bar{X}_n$ denote the mean of a random sample of size n from a distribution that is $N(\mu, \sigma^2)$. Find the limiting distribution of $\bar{X}_n$.

Or

- (b) Let X be $\chi^2(50)$. Approximate $\Pr(40 < X < 60)$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).

16. (a) State and prove Baye's theorem.

Or

- (b) State and prove Chebyshev's inequality.

17. (a) (i) Let the independent random variables X_1, X_2, X_3 have the same p.d.f $f(x) = 3x^2$, $0 < x < 1$, zero elsewhere. Find the probability that exactly two of these three variables exceed $\frac{1}{2}$.

- (ii) Let X be $b(2, p)$ and let Y be $b(4, p)$. If $\Pr(X \geq 1) = \frac{5}{9}$ find $\Pr(Y \geq 1)$.

Or

Page 6

Code No. : 6353



- (b) (i) If X is $\chi^2(5)$ determine the constant c and d so that $\Pr(c < X < d) = 0.95$ and $\Pr(X < C) = 0.025$.

- (ii) If X has a gamma distribution with $\alpha = 3$ and $\beta = 4$, find $\Pr(3.28 < X < 25.2)$.

18. (a) Let X_1, X_2 denote a random sample of size 2 from a distribution with p.d.f $f(x) = \frac{1}{2}, 0 < x < 2$, zero elsewhere. Find the joint p.d.f of X_1 and X_2 . Also find the distribution function and the p.d.f of $Y = X_1 + X_2$.

Or

- (b) Derive the p.d.f of F-distribution.

19. (a) Let $X_1 + X_2$ be two independent gamma random variables with parameters $\alpha_1 = 3, \beta_1 = 3$ and $\alpha_2 = 5, \beta_2 = 1$, respectively.

- (i) Find the m.g.f of $Y = 2X_1 + 6X_2$.
(ii) What is the distribution of Y ?

Or

Page 7

Code No. : 6353

- (b) (i) Determine the mean and variance of the mean $\bar{X}$ of a random sample of size 9 from a distribution having p.d.f $f(x) = 4x^3, 0 < x < 1$, zero elsewhere.

- (ii) Let X and Y be random variable with $\mu_1 = 1, \mu_2 = 4, \sigma_1^2 = 4, \sigma_2^2 = 6, \rho = \frac{1}{2}$. Find the mean and variance of $Z = 3X - 2Y$.

20. (a) Let T_n have a t-distribution with n degrees of freedom, $n = 1, 2, \dots$ with the distribution function $F_n(t)$. Prove that T_n has a limiting standard normal distribution.

Or

- (b) State and prove central limit theorem.

Page 8

Code No. : 6353

