(7 pages) **Reg. No. :** ....

Code No. : 7136 **Sub. Code : PMAM 44**

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2019.

Fourth Semester

Mathematics — Core

TOPOLOGY — II

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. The space R_K (K topology) is

   (a) T_4 (b) $T_{3\frac{1}{2}}$ but not T_4

   (c) T_3 but not $T_{3\frac{1}{2}}$ (d) T_2 but not T_3

2. Regular space is also known as

   (a) T_4 (b) $T_{2\frac{1}{2}}$

   (c) $T_{3\frac{1}{2}}$ (d) T_3

3. Which one of the following is normal

   (a) R_l

   (b) R_l^2

   (c) $S_\Omega \times \overline{S_\Omega}$

   (d) R^J, J is uncountable

4. A space X is completely regular then it is homeomorphic to a subspace of

   (a) $[0,1]^J$

   (b) $\mathbb{R}^n$ where n is a finite

   (c) $\mathbb{R}^J$

   (d) $(0,1)^J$ where n is a finite number and J is uncountable

5. Tietze extension theorem implies

   (a) The Urysohn Metrization theorem

   (b) Heine-Borel Theorem

   (c) The Urysohn Lemma

   (d) The Tychonof Theorem

6. Indicate the correct answer

(a) Subspace of a Normal space is normal

(b) Product of Normal spaces is normal

(c) R_l^2 is completely regular

(d) R_K is regular but not normal

7. Which one of the following is locally finite in R?

(a) $\{(n-1,n+1):n\in Z\}$

(b) $\left\{\left(0,\frac{1}{n}:n\in Z_+\right)\right\}$

(c) $\left\{\left(\frac{1}{n+1},\frac{1}{n}\right):n\in Z_+\right\}$

(d) $\{(x,x+1):x\in R\}$

8. Let $\mathcal{A}=\{(n-1,n+1):n\in Z\}$. Which of the following refine $\mathcal{A}$.

(a) $\left\{\left(n-\frac{1}{2},\ n+\frac{3}{2}\right):n\in Z_+\right\}$

(b) $\left\{\left(n+\frac{1}{2},\ n+\frac{3}{2}\right):n\in Z_+\right\}$

(c) $\left\{\left(n-\frac{1}{2},n+2\right):n\in Z_+\right\}$

(d) $\{(x,x+1):x\in R\}$

9. Which one of the following is false?

(a) Any set X with discrete topology is a Baire space

(b) The set of irrationals is not a Baire spaces

(c) [0, 1] is a Baire space

(d) Every locally compact space is a Baire space

10. Which of the following is not true?

(a) Every non empty open subset of the set of irrational numbers is of first category

(b) Open subspace of a Baire space is a Baire space

(c) Rationals as a subspace of real numbers is not a Baire space

(d) If $X=\bigcup_{n=1}^{\infty}B_n$ and X is a Baire space with $B_1\neq\phi$, then atleast one of $\overline{B}_n$ has nonempty interior

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Define $\mathbb{R}_K$ topological space. Prove that the space $\mathbb{R}_K$ is Hausdorff but not regular.

Or

(b) Prove that $\mathbb{R}_l^2$ is not a Lindeloff space.

12. (a) Show that the compact subset of a Hausdorff space is closed. Give an example of a closed subset of a Hausdorff space which is not compact.

Or

(b) Show that a compact Hausdorff space is normal.

13. (a) Prove that Tietze extension theorem implies the Urysohn Lemma.

Or

(b) State and prove imbedding theorem.

14. (a) Let A be a locally finite collection of subsets of X. Then prove that

(i) The collection $B = \{\overline{A} : A \in \mathcal{A}\}$ is locally finite

(ii) $\overline{\bigcup_{A \in \mathcal{A}} A} = \bigcup_{A \in \mathcal{A}} \overline{A}$

Or

(b) Define finite intersection property. Let X be a set and D be the set of all subsets of X that is maximal with respect to finite intersection property. Show that :

(i) $x \in \overline{A} \; \forall A \in D$ if and only if every neighborhood of x belongs to D.

(ii) Let $A \in D$. Then prove that $B \supset A \Rightarrow B \in D$

15. (a) Define second category space. Prove that any open subspace Y of a Baire space X is also a Baire space.

Or

(b) Define a Baire space. Whether Q the set of rationals as a space is a Baire space? What about if we consider Q as a subspace of real numbers space. Justify your answer

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 600 words.

16. (a) What are the countability axioms. Prove that the space $\mathbb{R}_L$ satisfies all the countability axioms but the second.

Or

(b) Prove that a normal space is a regular space but not conversely.

17. (a) Define a regular space, normal space and a second countable space. Prove that every regular second countable space is normal.

Or

(b) (i) Prove that every normal space is completely regular and completely regular space is regular.

(ii) Prove that product of completely regular spaces is completely regular.

18. (a) State and prove Tietze extension theorem.

Or

(b) Prove that every regular second countable space is metrizable.

19. (a) Let X be a metrizable space. If A is an open covering of X, then prove that there is an open covering ξ of X refining A that is countably locally finite.

Or

(b) State and prove Tychonoff theorem.

20. (a) State and prove Baire Category theorem.

Or

(b) Let X be a space; let (Y,d) be a metric space. Let $f_n : X \to Y$ be a sequence of continuous functions such that $f_n(x) \to f(x)$ for all $x \in X$, where $f : X \to Y$. If X is a Baire space, prove that the set of points at which f is continuous is dense in X.
