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Reg. No. : .....

Code No. : 7754 **Sub. Code : WMAE 11/ VMAE 11**

M.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2024.

First Semester

Mathematics

Elective I – GRAPH THEORY AND APPLICATIONS

(For those who joined in July 2023 only)

Time : Three hours Maximum : 75 marks

PART A — (15 × 1 = 15 marks)

Answer ALL questions.

Choose the correct answer :

1. A simple graph G is called ———— if $G \simeq G^c$.

   (a) Self-complementary

   (b) Non Adjacent

   (c) Disconnected

   (d) Isomorphic

2. A ———— of G is a complete subgraph of G.

   (a) Induced subgraph (b) Proper subgraph

   (c) Clique (d) Spanning subgraph

3. A 3 – regular graph is called a ————

   (a) Cubic graph (b) Spanning graph

   (c) Degree sequence (d) Hyper graph

4. A graph G is r – connected if $k(G) \geq r$. Also, G is r – edge connected if ————

   (a) $\lambda(G) \geq r$ (b) $\lambda(G) = r$

   (c) $\lambda(G) > r$ (d) $\lambda(G) < r$

5. A ———— of a graph is a maximal nonseparable of G. If G has no cut vertex.

   (a) Path (b) Cycle

   (c) Block (d) Subgraph

6. An edge e is a ———— of G if $\{e\}$ is an edge cut of G.

   (a) Vertex cut (b) Cut edge

   (c) Cut vertex (d) Edge cut

7. Let G be a connected graph, then the diameter of G is defined as ——— and is denoted by $diam(G)$.

(a) $\max\{d(u,v) : u,v \in V(G)\}$

(b) $\min\{d(u,v) : u,v \in V(G)\}$

(c) $\max\{d(u,v) : u,v \notin V(G)\}$

(d) $\min\{d(u,v) : u,v \notin V(G)\}$

8. The ——— of a vertex u of T is the maximum number of edges in any branch at u.

(a) Centroid (b) Weight

(c) Branch (d) Eccentricity

9. A vertex v of G is called ——— if $e(v) = r(G)$.

(a) Diameter (b) Radius

(c) Central vertex (d) Eccentricity

10. A subset M of the edge set E of a loopless graph G is called ——— if no two edges of M are adjacent in G.

(a) Independent (b) Covering

(c) Regular (d) Spanning

11. A ——— is G is a set of independent edges.

(a) Matching (b) Spanning

(c) Complement (d) Connected

12. A ——— of a graph G is a spanning subgraph of G.

(a) Factor (b) Unsaturated

(c) Matching (d) Decomposition

13. An ——— in a graph G is a spanning trail in G that contains all the edges of G.

(a) Hamiltonian walk (b) Euler trail

(c) Euler walk (d) Hamiltonian trail

14. A graph G is called ——— if it has a spanning path of G.

(a) Traceable (b) Hamiltonian

(c) Euler (d) Closure

15. The ——— of a graph G is the minimum number colors needed for a proper vertex coloring of G.

(a) Chromatic number (b) Boundary

(c) Spectrum (d) Decomposition

PART B — (5 × 4 = 20 marks)

Answer ALL the questions, choosing either (a) or (b).

16. (a) If G is simple and $\delta \geq \frac{n-1}{2}$, then prove that G is connected.

Or

(b) If the simple graphs G_1 and G_2 are isomorphic then prove that $L(G_1)$ and $L(G_2)$ are isomorphic.

17. (a) Show that a vertex v of a connected graph G with atleast three vertices is a cut vertex of G if and only if there exists vertices u and w of G distinct from v such that v is in every $u-w$ path in G.

Or

(b) Prove that a connected simple graph G is 3-edge connected if and only if every edge of G is the exact intersection of the edge sets of two cycles of G.

18. (a) Show that a simple graph is a tree if and only if any two distinct vertices are connected by a unique path.

Or

(b) Prove that a connected graph G with at least two vertices is a tree if and only if its degree sequence $(d_1, d_2, ..., d_n)$ satisfies the condition: $\sum_{i=1}^{n} d_i = 2(n-1)$ with $d_i > 0$ for each i.

19. (a) Show that a $k(\geq 1)$ – regular bipartite graph is 1-factorable.

Or

(b) Let G be a connected graph of even order n. If G is claw-free, then prove that G has a 1-factor.

20. (a) Let G be a simple graph with $n \geq 3$ vertices. If for every pair of nonadjacent vertices u, v of G, $d(u) + d(v) \geq n$, then prove that G is Hamiltonian.

Or

(b) If G is k-critical, then prove that $\delta(G) \geq n-1$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b)

21. (a) If G is a line graph, then prove that $K_{1,3}$ is a forbidden subgraph of G.

Or

(b) Let G and G' be simple connected graphs with isomorphic line graphs. Then prove that G and G' are isomorphic unless one of them is $K_{1,3}$ and the other is K_3.

22. (a) Show that for any loopless connected graph $G, k(G) \leq \lambda(G) \leq \delta(G)$.

Or

(b) Prove that the connectivity and edge connectivity of a simple cubic graph G are equal.

23. (a) If e is not a loop of a connected graph, then prove that $\tau(G) = \tau(G-e) + \tau(G \circ e)$.

Or

(b) Show that every tree has a center consisting of either a single vertex or two adjacent vertices.

24. (a) Prove that a matching M of a graph G is maximum if and only if G has no M-argumenting path.

Or

(b) Show that for any graph G for which $\delta > 0, \alpha' + \beta' = n$.

25. (a) Prove that for a nontrivial connected graph G, the following statements are equivalent:

(i) G is Eulerian.

(ii) The degree of each vertex of G is even positive integer.

(iii) G is an edge-disjoint union of cycles.

Or

(b) Show that for any simple graph G, $2\sqrt{n} \leq \chi + \chi^c \leq n+1$ and $n \leq \chi\chi^c \leq \left(\frac{n+1}{2}\right)^2$.
