(7 pages)

Reg. No. : ....

Code No. : 20653 E **Sub. Code : EMMA 12**

B.Sc. (CBCS) DEGREE EXAMINATION,
NOVEMBER 2023.

First Semester

Mathematics – Core

DIFFERENTIAL CALCULUS

(For those who joined in July 2023 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. $D^n(ax+b)^{-1} =$ ____________.

   (a) $(-1)^n a^n (ax+b)^{-n-1}$

   (b) $(-1)^n n! a^n (ax+b)^{-n-1}$

   (c) $(-1)^n a^n (ax+b)^{-n}$

   (d) $(-1)^n n! a^n (ax+b)^{-n}$

2. $D^n(\cos x) =$ ____________.

   (a) $\cos\left(\frac{n\pi}{2}+x\right)$ (b) $\sin\left(\frac{n\pi}{2}+x\right)$

   (c) $\cos\frac{n\pi}{2}x$ (d) $\sin\frac{n\pi}{2}x$

3. If $z=f(u)$ and $u=\varphi(x,y)$ $(x,y$ are independent variables), then $\frac{\partial z}{\partial x} =$ ____________.

   (a) $\frac{\partial z}{\partial u}\frac{du}{dx}$ (b) $\frac{\partial z}{\partial u}\frac{du}{dy}$

   (c) $\frac{dz}{du}\frac{\partial u}{\partial x}$ (d) $\frac{\partial z}{\partial u}\frac{\partial u}{\partial x}+\frac{\partial z}{\partial u}\frac{\partial u}{\partial y}$

4. If $x^3+y^3=3axy$, $\frac{dy}{dx} =$ ____________.

   (a) $\frac{x^2-ay}{y^2-ax}$ (b) $\frac{y^2-ax}{x^2-ay}$

   (c) $\frac{ax-y^2}{x^2-ay}$ (d) $\frac{ay-x^2}{y^2-ax}$

5. $f(x,y) = \frac{x^3 - y^3}{x + y}$ is a homogeneous function of degree ______________.

(a) 2 (b) 4

(c) 3 (d) 1

6. If $f(x,y)$ is a homogeneous function of degree n, ______________.

(a) $f(\lambda x, y) = \lambda^n f(x,y)$

(b) $f(x, \lambda y) = \lambda^n f(x,y)$

(c) $f(\lambda x, \lambda y) = \lambda^n f(x,y)$

(d) $f(\lambda x, \lambda y) = \lambda^{2n} f(x,y)$

7. The envelope of the family of curves $\frac{x\cos\alpha}{a} + \frac{y\sin\alpha}{b} = 1$ where α is the parameter and a and b are constants is ______________.

(a) a circle (b) an ellipse

(c) a straight line (d) a parabola

8. The evolute of a curve is the ______________ of the normals to the curve.

(a) involute (b) evolute

(c) envelope (d) normal

9. The radius of curvature at the point $x = \pi/2$ on the curve $y = \sin x$ is ______________.

(a) 1 (b) –1

(c) 0 (d) 2

10. The centre of curvature of the curve $xy = c^2$ at the point (c, c) is ______________.

(a) (c, c) (b) $(2c, c)$

(c) $(c, 2c)$ (d) $(2c, 2c)$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If $y = \log(ax + b)$, find y_n.

Or

(b) If $xy = ae^x + be^{-x}$, prove that $x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - xy = 0$.

12. (a) If $u = \log\frac{x^2 + y^2}{xy}$, prove that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$.

Or

(b) Find $\frac{du}{dt}$ where $u = x^2 + y^2 + z^2$, $x = e^t$, $y = e^t \sin t$, $z = e^t \cos t$.

13. (a) Verify Euler's theorem for the function $u = x^3 - 2x^2y + 3xy^2 + y^3$.

Or

(b) If $u = \sin\left(\frac{x^2+y^2}{x+y}\right)$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{x^2+y^2}{x+y}\cos\left(\frac{x^2+y^2}{x+y}\right)$.

14. (a) Find the envelope of the family of circles $x^2 + y^2 - 2ax\cos\theta - 2ay\sin\theta = c^2$ (θ - parameter).

Or

(b) Find the envelope of the family of curves $y = m^2x + am$ (m-parameter).

15. (a) Find the radius of curvature of the curve $r = a(1-\cos\theta)$.

Or

(b) Find the centre of curvature of the curve $y = x\log x$ at the point where $y_1 = 0$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Find (i) $D^n(\cos x \cos 2x \cos 3x)$

(ii) $D^n\left(\log\frac{2x+3}{3x+2}\right)$.

Or

(b) If $y = \sin(m\sin^{-1}x)$, prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2-n^2)y_n = 0$.

17. (a) If $V = (x^2+y^2+z^2)^{-1/2}$, prove that $\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} = 0$.

Or

(b) If $x = e^{-t}\cos\theta$, $y = e^{-t}\sin\theta$, prove that $\frac{\partial t}{\partial x} = \frac{-x}{x^2+y^2}$ and $\frac{\partial \theta}{\partial x} = \frac{-y}{x^2+y^2}$.

18. (a) State and prove Euler's theorem.

Or

(b) If $u = \tan^{-1}\left(y^2/x\right)$, prove that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \sin 2u \sin^2 u$.

19. (a) Find the envelope of the family of straight lines $\frac{x}{a}+\frac{y}{b}=1$ where $a^2+b^2=k^2$ and k is a constant.

Or

(b) Prove that the envelope of the family of ellipses $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ $(a^2+b^2=c^2)$ are $x+y=\pm c$ and $x-y=\pm c$.

20. (a) Find the radius of curvature at the point 't' of the curve $x=a(\cos t+t\sin t)$; $y=a(\sin t-t\cos t)$.

Or

(b) Show that the evolute of the cycloid $x=a(\theta-\sin\theta)$; $y=a(1-\cos\theta)$ is another cycloid.

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