(7 pages) **Reg. No. :** .....

Code No. : 10726 E **Sub. Code : EMMA 22**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2024.

Second Semester

Mathematics — Core

INTEGRAL CALCULUS

(For those who joined in July 2023 onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer :

1. $\int_0^{\frac{\pi}{2}} x\left(1-x^2\right)^{\frac{1}{2}}\, dx =$ _____

(a) 0 (b) $\frac{\pi}{2}$

(c) $\frac{\pi}{3}$ (d) $\frac{1}{3}$

2. $\int_0^{\frac{\pi}{4}} \tan^3 x\, dx =$ _____

(a) 0 (b) 1

(c) $\frac{1}{2}(1-\log 2)$ (d) 3

3. $\int_0^3 \int_1^2 xy\,(x+y)\, dydx =$ _____

(a) 24 (b) 20

(c) 12 (d) 10

4. $\int_0^2 \int_0^x xy^2\, dy\, dx =$ _____

(a) $\frac{1}{15}$ (b) $\frac{32}{15}$

(c) $\frac{16}{15}$ (d) $\frac{8}{15}$

5. If $x = r\cos\theta$, $y = r\sin\theta$, then $J =$ _____

(a) 0 (b) 1

(c) r (d) θ

6. $\int_0^1\int_0^1\int_0^1 xyz\,dx\,dy\,dz =$ _____

(a) $\frac{1}{2}$ (b) $\frac{1}{8}$

(c) $\frac{1}{6}$ (d) 3

7. $\int_1^\infty \frac{dx}{x^2} =$ _____

(a) 0 (b) 1

(c) -1 (d) ∞

8. $\beta(1, 1) =$ _____

(a) 0 (b) 2

(c) $\sqrt{\pi}$ (d) 1

9. The entire area of the lemniscates of Bernoulli $r^2 = a^2 \cos 2\theta$ is

(a) a^2 (b) $2a^2$

(c) $4a$ (d) $2a$

10. The moment of inertia of a circular lamina of radius a about any diameter is _____

(a) ma^2 (b) $\frac{ma^2}{2}$

(c) $\frac{ma^2}{4}$ (d) ma

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions choosing either (a) or (b).
Each answer should not exceed 250 words.

11. (a) Find the reduction formula for $\int \sin^n x\,dx$.

Or

(b) Evaluate ; $\int x^3 \cos 2x\,dx$.

12. (a) Compute $\iint (x^2 + y^2)\,dx\,dy$ over the region for which x, y are each ≥ 0 and $x + y \leq 1$.

Or

(b) Estimate $\iint r\sqrt{a^2 - r^2}\,dr\,d\theta$ over the upper half of the circle $r = a\cos\theta$.

13. (a) Find the area of the cardioid $r = a(1+\cos\theta)$.

Or

(b) Obtain the area of the portion of the cone $x^2 + y^2 = 4z^2$ lying above the $X \circ Y$ plane and inside the cylinder $x^2 + y^2 = 3y$.

14. (a) Evaluate :

(i) $\int_0^{\infty} e^{-x^2}\, dx$

(ii) $\int_0^{\frac{\pi}{2}} \sqrt{\tan\theta}\, d\theta$.

Or

(b) Prove :

(i) $\Gamma(n+1) = n\Gamma(n)$

(ii) If n is a positive integer, then $\Gamma(n+1) = n!$.

15. (a) Find the area enclosed by the parabola $y = x^2$ and the straight line $2x - y + 3 = 0$.

Or

(b) Obtain the centroid of the arc and the sector of a circle.

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).
Each answer should not exceed 600 words.

16. (a) Explain the method of finding reduction formula for $\int \sin^m x \cos^n x\, dx$.

Or

(b) Find a reduction formula for $\int \sec^n x\, dx$. Also evaluate $\int \sec^3 x\, dx$.

17. (a) By changing into polar co-ordinates, evaluate the integral $\int_0^{2a} \int_0^{\sqrt{2ax - x^2}} (x^2 + y^2)\, dx\, dy$.

Or

(b) Evaluate the value of $\int_0^{\infty} \int_x^{\infty} \frac{e^{-y}}{y}\, dx dy$ by changing the order of integration.

18. (a) Compute $\iiint xyz\, dx\, dy dz$ taken through the positive octant of the sphere $x^2 + y^2 + z^2 = a^2$.

Or

(b) Prove : $\frac{\partial(u, v)}{\partial(x, y)} \cdot \frac{\partial(x, y)}{\partial(\xi, \eta)} = \frac{\partial(u, v)}{\partial(\xi, n)}$.

19. (a) Derive the relation between Beta and Gamma functions.

Or

(b) If $x, y, z \geq 0$ and $x^2 + y^2 + z^2 \leq 1$, then show that $\iiint \frac{dxdydz}{\left(1 - x^2 - y^2 - z^2\right)^{\frac{1}{2}}} = \frac{\pi^2}{8}$.

20. (a) Find the length of one loop of the curve $3ay^2 = x\left(x - a\right)^2$.

Or

(b) Obtain the moment of inertia of a solid sphere about the diameter.
