(8 pages) **Reg. No. :** ....

Code No. : 6353 **Sub. Code : HMAM 13**

M.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2014.

First Semester

Mathematics

PROBABILITY AND STATISTICS

(For those who joined in July 2012 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. What is the value of $P(C_1/C_1)$?

   (a) 1 (b) C_1

   (c) 0 (d) ½

2. If $P(C_1)=\frac{3}{8}$ and $P(C_2/C_1)=\frac{5}{7}$ then the value of $P(C_1 \cap C_2)$ is

   (a) $\frac{3}{7}$ (b) $\frac{5}{8}$

   (c) $\frac{56}{15}$ (d) $\frac{15}{56}$

3. Let X has a binomial distribution. If mean = $\frac{7}{2}$ and variance = $\frac{7}{4}$, what is the value of n?

   (a) 7 (b) 4

   (c) 2 (d) $\frac{1}{2}$

4. Suppose that X has a Poisson distribution with $\mu = 2$. The value of variance = ————————.

   (a) 4 (b) 2

   (c) 1 (d) 0

5. A function of one or more random variables that does not depend upon any unknown parameter is called ————————.

   (a) distribution (b) sampling

   (c) density function (d) statistic

6. Let X have a p.d.f. $f(x) = (½)^x$, $x = 1,2,3,\cdots$, zero elsewhere. The p.d.f. of $Y = X^3$ is given by $g(y) =$ ————, $y = 1,8,27,\cdots$, zero elsewhere.

(a) $\frac{1}{y^3}$ (b) $\left(\frac{1}{2}\right)^y$

(c) $\left(\frac{1}{3}\right)^y$ (d) $\left(\frac{1}{2}\right)^{\sqrt[3]{y}}$

7. If $X_1, X_2, \cdots, X_n$ are the random samples with MGF $M(t)$, then the MGF of $\sum_{i=1}^{n} \frac{X_i}{n}$ is ————.

(a) $[M(t)]^n$ (b) $M\left(\frac{t^n}{n}\right)$

(c) $\left[M\left(\frac{1}{t}\right)\right]^n$ (d) $\left[M\left(\frac{t}{n}\right)\right]^n$

8. Let X_1, X_2, X_3, X_4 denote a random sample from a distribution that is $N\left(\frac{2}{3}, \frac{1}{18}\right)$. The mean and variance of the random variable $Y = X_1 + X_2 + X_3 + X_4$ is ————.

(a) $\frac{8}{3}, \frac{1}{18}$ (b) $\frac{8}{3} \cdot \frac{2}{9}$

(c) $\frac{3}{8}, \frac{9}{2}$ (d) $\frac{2}{3}, \frac{1}{18}$

9. $\lim_{n \to \infty} F_n = F(x)$ at all points and for which $F(x)$ is continuous, here $F(x)$ is called as ———— distribution.

(a) limiting (b) diverges

(c) sequence (d) none of the above

10. A sequence of random variables $X_1, X_2, \cdots$ converges in probability to a random variable X if for every $\in > 0$ $\lim_{n \to \infty} \Pr(|X_n - X| < \in) =$ ————.

(a) 0 (b) 1

(c) ∞ (d) $-\infty$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b), each answer should not exceed 250 words.

11. (a) A hand of 5 cards is to be dealt at random without replacement from an ordinary deck of 52 playing cards. Find the conditional probability of an all spade hand (C_2), relative to the hypothesis that these are atleast 4 spades in the hand (C_1).

Or

(b) If $P(C_1 > 0)$ and if $C_2, C_3, C_4 \cdots$ are mutually disjoint sets, show that $P(C_2 \cup C_3 \cup \cdots | C_1) = P(C_2 | C_1) + P(C_3 | C_1) + \cdots$.

12. (a) If the m.g.f. of a random variable X is $\left(\frac{1}{3}+\frac{2}{3}e^t\right)^5$, find $\Pr(X = 2 \text{ or } 3)$.

Or

(b) Let the independent random variables X_1, X_2, X_3 have the same pdf $f(x) = 3x^2$, $0 < x < 1$, zero elsewhere. Find the probability that exactly two of these three variables exceed $\frac{1}{2}$.

13. (a) Let X_1, X_2, X_3 be a random sample of size 3 from a distribution that is $N(6,4)$. Determine the probability that the largest sample observation exceeds 8.

Or

(b) Let X have the p.d.f. $f(x) = \frac{1}{3}$, $x = 1, 2, 3$, zero elsewhere. Find the p.d.f. of $Y = 2X + 1$.

14. (a) Let X_1 and X_2 be independent with normal distributions $N(6,1)$ and $N(7,1)$, respectively. Find $\Pr(X_1 > X_2)$.

Or

(b) Let X_1, X_2, X_3, X_4 be four independent identically distributed random variables having the same p.d.f. $f(x) = 2x$, $0 < x < 1$ zero elsewhere. Find the mean and variance of the sum Y of these four random variables.

15. (a) Let $\overline{X}_n$ denote the mean of a random sample of size n from a distribution that is $N(\mu, \sigma^2)$. Find the limiting distribution of $\overline{X}_n$.

Or

(b) Let Z_n be $\chi^2(n)$ and let $W_n = \frac{Z_n}{n^2}$. Find the limiting distribution of W_n.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b), each answer should not exceed 600 words.

16. (a) A bowl contains 8 chips. Three of the chips are red and 5 are blue. Four chips are to be drawn successively at random and without replacement.

(i) Compute the probability that the colors alternate.

(ii) Compute the probability that the first blue chip appears on the third draw.

Or

(b) State and prove Chebyshev's inequality.

17. (a) Find the m.g.f. of a Poisson distribution. Also find its mean and variance.

Or

(b) (i) If the random variable X is $N(\mu, \sigma^2)$, $\sigma^2 > 0$, then the random variable $W = \frac{X - \mu}{\sigma}$ is $N(0, 1)$.

(ii) Let X be $N(2,25)$. Find $\Pr(0 < X < 10)$ and $\Pr(-8 < X < 1)$.

18. (a) Let X_1, X_2 denote a random sample of size 2 from a distribution with p.d.f. $f(x) = 1$, $0 < x < 1$, zero elsewhere. Find the joint p.d.f. of X_1 and X_2. Also find the distribution function and the p.d.f. of $Y = \frac{X_1}{X_2}$.

Or

(b) Derive p.d.f. of t-distribution.

19. (a) Let $F_n(y)$ denote the distribution function of a random variable y_n whose distribution depends upon the positive integer n. Let C denote a constant which does not depend upon n. Prove that the sequence Y_n, $n = 1, 2, 3, \cdots$, converges in probability to c iff the limiting distribution of Y_n is degenerate at $y = c$.

Or

(b) State and prove Central limit theorem.

20. (a) Let X_1, X_2 be two independent gamma random variables with parameters $\alpha_1 = 3$, $\beta_1 = 3$ and $\alpha_2 = 5$, $\beta_2 = 1$, respectively.

(i) Find the m.g.f. of $Y = 2X_1 + 6X_2$

(ii) What is the distribution of Y?

Or

(b) Prove that the distribution of $\frac{ns^2}{\sigma^2}$ is $\chi^2(n-1)$
