(7 pages) **Reg. No. :** ........................................

**Code No. : 20468 E** **Sub. Code : CAMA 11**

B.Sc. (CBCS) DEGREE EXAMINATION,
NOVEMBER 2021.

First Semester

Mathematics – Allied

ALGEBRA AND DIFFERENTIAL EQUATIONS

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer.

1. The equation $ax^5 + bx^4 + cx^3 + dx^2 + ex + f$ can have ———— roots and no more.

   (a) 5

   (b) 3

   (c) 4

   (d) none of these

2. If $\alpha, \beta, \gamma$ are the roots of the equation $x^4 + px^3 + qx^2 + rx + s = 0$ then $\sum \alpha\beta\gamma =$ ————————.

(a) $-p$ (b) $q$

(c) $-r$ (d) $s$

3. How many imaginary roots will occur for the equation $x^7 - 3x^4 + 2x^3 - 1 = 0$ ?

(a) atmost four

(b) exactly four

(c) atleast four

(d) none of these

4. To remove the fractional co-efficients from $x^3 - \frac{1}{4}x^2 + \frac{1}{3}x - 1 = 0$ multiply the roots by ————————.

(a) 4 (b) 3

(c) –3 (d) 12

5. The characteristics equation of $A = \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}$ is

(a) $\lambda^2 - 2\lambda - 3 = 0$ (b) $\lambda^2 - 2\lambda + 3 = 0$

(c) $\lambda^2 - 3\lambda + 2 = 0$ (d) $\lambda^2 - 5\lambda + 6 = 0$

6. The eigen values of $\begin{pmatrix} 1 & 0 \\ 0 & 8 \end{pmatrix}$ are

(a) $-1, -8$ (b) $1, -8$

(c) $1, 8$ (d) $-1, 8$

7. The Clairaut's equation is ————————.

(a) $y = cx + f(c)$

(b) $y = px + f(p)$

(c) $\frac{dy}{dx} = \left\{p + x\frac{dp}{dx}\right\} + f(p)\frac{dp}{dx}$

(d) none of these

8. The partial differential equation from $z = f(y/x)$ is ————————.

(a) $px + qy = 0$ (b) $px - qy = 0$

(c) $qx - py = 0$ (d) none of these

9. $L(1) =$ ————————

(a) $\frac{1}{s}$ (b) $\frac{1}{s^2}$

(c) $-\frac{1}{s}$ (d) $-\frac{1}{s^2}$

10. $L^{-1}\left[\frac{1}{s-a}\right] =$ ____________.

(a) 1 (b) $x$

(c) $e^{ax}$ (d) None

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Solve the equation $x^4 - 5x^3 + 4x^2 + 8x - 8 = 0$ given that one root is $1-\sqrt{5}$.

Or

(b) Solve the equation $6x^3 - 11x^2 + 6x - 1 = 0$ whose roots are in harmonic progression.

12. (a) Diminish the roots of $x^4 - x^3 - 10x^2 + 4x + 24 = 0$ by 2 and hence solve the original equation.

Or

(b) Solve the equation $x^3 - 4x^2 - 3x + 18 = 0$ given that two of its roots are equal.

13. (a) Find the inverse of the matrix $\begin{bmatrix} 2 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$.

Or

(b) Find the eigen value and eigen vectors of $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$.

14. (a) Form a partial differential equation by eliminating the arbitrary function $\phi$ from $\phi(x+y+z,\ x^2+y^2-z^2)=0$.

Or

(b) Eliminate the arbitrary constants $a, b$ and $c$ from $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$.

15. (a) (i) Prove that $L[x^n]=\frac{\lceil(n+1)}{s^{n+1}}$

(ii) If $L[f(x)]=F(s)$ then prove that $L[x\,f(x)]=\frac{d}{ds}[F(s)]$.

Or

(b) Find $L\left(\frac{1-\cos x}{x}\right)$.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Show that the roots of the equation $px^3 + qx^2 + rx + s = 0$ are in arithmetic progression if $2q^3 + 27p^2s = 9pqr$.

Or

(b) Solve the equation $6x^5 - x^4 - 43x^3 + 43x^2 + x - 6 = 0$.

17. (a) Obtain by Newton's method the root of the equation $x^3 - 3x + 1 = 0$ which lies between 1 and 2.

Or

(b) Find by Horner's method the positive root of $x^3 - 3x + 1 = 0$ correct to three decimal places.

18. (a) Find the eigen values and eigen vectors of $\begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$.

Or

(b) Verify Cayley-Hamilton theorem for $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$.

19. (a) Solve : $xyp^2 + (3x^2 - 2y^2)\,p - 6xy = 0$.

Or

(b) Solve : $x^2\,(y-z)\,p + y^2\,(z-x)\,q = z^2(x-y)$.

20. (a) Find $L^{-1}\left[\dfrac{cs+d}{(s+a)^2+b^2}\right]$.

Or

(b) Find : $L^{-1}\left[\dfrac{5s+3}{(s-1)(s^2+2s+5)}\right]$.

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