(6 pages)

Reg. No. : ....

Code No. : 30579 E **Sub. Code : SMMA 62**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2020.

Sixth Semester

Mathematics — Core

NUMBER THEORY

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer

1. The sum of $41+42+\ldots+78$ is

   (a) 3081 (b) 2261

   (c) 2061 (d) 1661

2. If n is a given positive integer, and $r \le n$ is also a positive integer, than the value of $nc_r + nc_{r-1}$

   (a) $n+1c_r$ (b) $n+1c_{r+1}$

   (c) nc_r (d) nc_{r+1}

3. gcd $(-8, -36) =$ ____________

(a) -8 (b) -4

(c) 4 (d) 8

4. For any interger $k \neq 0$, gcd$(ka, kb) = ?$

(a) K.gcd(a,b) (b) $|K|$gcd(a,b)

(c) gcd(a,b) (d) k^2gcd(a,b)

5. The number of odd prime less than 30 is

(a) 8 (b) 9

(c) 10 (d) 11

6. According to division algorithm, every positive even integer can be uniquely written as

(a) $4n+1$ (b) $4n+3$

(c) $4n(or)4n+2$ (d) None of these

7. The congruence $6x \equiv$ ____________ $(\bmod 21)$ has solutions.

(a) 3 (b) 2

(c) 6 (d) 8

8. In ISBN, the tenth digit a_{10} is given by ______________

(a) $\sum_{k=1}^{9} Ka_k \pmod{11}$

(b) $\sum_{k=1}^{9} a_k \pmod{11}$

(c) $\sum_{k=1}^{9} (K+1)a_k \pmod{11}$

(d) $\sum_{k=1}^{10} Ka_k \pmod{11}$

9. The value of $\phi(225)$ is

(a) 15 (b) 45

(c) 75 (d) 120

10. If P is an odd prime find the remainder when $1^{p-1} + 2^{p-1} + \ldots + (p-1)^{p-1}$ is divided by P.

(a) 1 (b) 2

(c) $\frac{p-1}{2}$ (d) $p-1$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) (i) Prove that the sum of first n natural numbers is a triangular number.

(ii) The sum of any 2 consecutive triangular numbers is a perfect square.

Or

(b) For $n \geq 2$, Find the value of $\binom{2}{2}+\binom{3}{2}+\binom{4}{2}+\ldots+\binom{n}{2}+\binom{n+1}{3}$.

12. (a) Let a,b,c be integers no two of which are zero. Show that $d = \gcd(a,b,c)$

$d = \gcd(\gcd(a,b),c)$

$= \gcd(a,b,c) = (a, \gcd(b,c))$

Or

(b) Prove that $\gcd(a, b).1\mathrm{cm}(a, b) = ab$ for positive integers.

13. (a) Find the prime factorization of

(i) 10140

(ii) 36000

Or

(b) If P_n is the n^{th} prime number, then prove that $P_n \leq 2^{2^{n-1}}$.

14. (a) Calculate $41^{65} \equiv 6 \pmod{7}$.

Or

(b) Solve the linear congruence $18x \equiv 30 \pmod{42}$

15. (a) Explain about the converse of the Fermat's theorem by giving an example.

Or

(b) If P is a prime, prove that for any integer $a, P \mid a^p + (p-1)!$ and $P \mid (p-1)!\, a^p + a$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Establish the binomial theorem.

Or

(b) (i) Prove that

$$1.2 + 2.3 + 3.4 + \cdots + n(n+1) = \frac{n(n+1)(n+2)}{3}, \forall n \geq 1.$$

(ii) State and prove the second principle of finite induction.

17. (a) State and prove Euclidean Algorithm.

Or

(b) (i) If a/b and a/c, prove that $a \mid (bx + cy), x, y \in Z$.

(ii) State and prove Division Algorithm.

18. (a) State and prove fundamental theorem of arithmetic.

Or

(b) (i) Show that there are infinite number of primes.

(ii) Show that the number $\sqrt{2}$ is irrational

19. (a) (i) State and prove Chinese Remainder theorem.

(ii) Solve $x \equiv 2 \pmod 3), x \equiv 3 \pmod 5, x \equiv 2 \pmod 7$

Or

(b) (i) If $a \equiv b \pmod m$ and $f(x)$ is a polynomial coefficient, show that $f(a) \equiv f(b) \pmod m$.

(ii) Using congruences prove that the Fermat's number $F_5 = 2^{32} + 1$ is not a prime.

20. (a) Show that $a^{21} \equiv a \pmod{15}$.

Or

(b) State and prove Wilson's theorem.