(8 pages) **Reg. No. :** ....

Code No. : 5674 **Sub. Code : ZMAM 25**

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2022

Second Semester

Mathematics — Core

RESEARCH METHODOLOGY AND STATISTICS

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. Which one of the following is not true

   (a) the title page of a research report should not be numbered

   (b) the pages of the main body of the text are numbered with Arabic numerals

   (c) the page of preliminary sections should be numbered using Roman numerals

   (d) the page of preliminary sections should be numbered using Arabic numerals

2. Typically, abstracts are between ———— and ———— words in length.

   (a) 10 and 20 (b) 250 and 300

   (c) 1200 and 1500 (d) 5 and 10

3. Let the joint p.d.f of X_1 and X_2 be

$$f(x_1, x_2) = \frac{x_1 + x_2}{21}, \; x_1 = 1,2,3, \; x_2 = 1,2$$
$$= 0, \quad \text{elsewhere}$$

   Then $P_r(X_1 = 3)$ is

   (a) $\frac{3}{7}$ (b) $\frac{4}{21}$

   (c) $\frac{5}{21}$ (d) $\frac{1}{7}$

4. The m.g.f. $M(t_1, t_2)$ of the joint distribution of X and Y is

   (a) $E(t_1 X + t_2 Y)$ (b) $E(e^{t_1 X \cdot t_2 Y})$

   (c) $E(e^{t_1 X + t_2 Y})$ (d) $E(t_1 X + t_1 Y)$

5. If $(1-2t)^{-6}$, $t < \frac{1}{2}$ is the m.g.f. the random variable X, then X is

   (a) $\chi^2(6)$ (b) $\chi^2(3)$

   (c) $\chi^2(-6)$ (d) $\chi^2(12)$

6. For a gamma distribution σ^2 is

(a) $\alpha\beta^2$ (b) $\alpha\beta$

(c) $\alpha^2\beta$ (d) $\frac{\alpha}{\beta^2}$

7. If $x_1 = 2y_1 + y_2$, $x_2 = y_2$ then the value of J is

(a) –2 (b) 2

(c) 1/2 (d) –1/2

8. If W is $n(0,1)$ and V is $\chi^2(r)$ and if W and V are stochastically independent, the which one of the following is at distribution

(a) $\frac{W}{\sqrt{V/r}}$ (b) $\frac{W}{\sqrt{Vr}}$

(c) $\frac{W}{(V/r)}$ (d) $\sqrt{\frac{W}{V/r}}$

9. Let X_1 and X_2 be stochastically independent with normal distribution $n(\mu_1, \sigma_1^2)$ and $n(\mu_2, \sigma_2^2)$ respectively. Then $Y = X_1 - X_2$ is

(a) $n(\mu_1 - \mu_2, \sigma_1^2 - \sigma_2^2)$

(b) $n(\mu_1 - \mu_2, \sigma_1^2 + \sigma_2^2)$

(c) $n(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2)$

(d) $n(\mu_1\mu_2, \sigma_1^2.\sigma_2^2)$

10. If X_1, X_2, X_n denote a random sample from a distribution with m.g.f. $M(t)$. Then m.g.f. of $\sum_1^n \frac{X_i}{n}$ is

(a) $M(t)^n$ (b) $M\left(\frac{t}{n}\right)^n$

(c) $M(t^n)$ (d) $M(tn)^n$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Write an abstract for your research project (you can choose your own topic).

Or

(b) What is methodology and why is it important?

12. (a) If the random variable X_1 and X_2 have the joint p.d.θ $f(x_1, x_2) = 2e^{-x_1 - x_2}$, $0 < x_1 < x_2$, $0 < x_2 < \infty$ and zero elsewhere, prove that X_1 and X_2 are stochastically dependent.

Or

(b) Let X_1 and X_2 have the joint p.d.f. $f(x_1, x_2) = 2$, $0 < x_1 < x_2 < 1$, zero elsewhere. Find the marginal probability density functions and the conditional p.d.f. of X_1 given $X_2 = x_2$, $0 < x_2 < 1$.

13. (a) Let X have a gamma distribution with $\alpha = r/2$, when r is a positive integer and $\beta > 0$. Define $Y = 2X/\beta$. Find the p.d.f. of Y.

Or

(b) If the random variable X is $N(\mu, \sigma^2)$, $\sigma^2 > 0$, prove that the random variable $V = \frac{(X-\mu)^2}{\sigma^2}$ is $\chi^2(1)$.

14. (a) Let X have the p.d.f. $f(x) = 1$, $0 < x < 1$, zero elsewhere. Show that the random variable $Y = -2 \log X$ has a chi-square distribution with 2 degree of freedom.

Or

(b) Let X have the p.d.f. $f(x) = x^2/9$, $0 < x < 3$, zero elsewhere. Find the p.d.f. of $Y = X^3$.

15. (a) Let X_1 and X_2 be stochastically independent with normal distribution $N(\mu_1, \sigma_1^2)$ and $N(\mu_2, \sigma_2^2)$ respectively. Let $Y = X_1 - X_2$. Using m.g.f. technique, find the p.d.f. of Y.

Or

(b) Let $\overline{X}$ denote the mean of a random sample of size 128 from a gamma distribution with $\alpha = 2$ and $\beta = 4$. Approximate $\Pr(7 < \overline{X} < 9)$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) (i) Explain the importance of Literature review.

(ii) Write a short note on plagiarism.

Or

(b) What are the different components of a research project? Explain your answer.

17. (a) Show that $E[E(X_2 \mid X_1)] = E(X_2)$ and $\text{var}[E(X_2 \mid X_1) \leq \text{var}\, X_2$.

Or

(b) Show that X_1 and X_2 are independent if and only if $M(t_1, t_2) = M(t_1, 0)\ M(0, t_2)$.

18. (a) Define a gamma distribution and obtain its m.g.f. mean and variance.

Or

(b) Compute the measures of Skewness and Kurtosis of a gamma distribution with parameters α and β.

19. (a) Let X_1, X_2, X_3 denote a random sample of size 3 from a standard normal distribution. Let Y all note the statistic that is the sum of the squares of the sample observations. Find the p.d.f. of Y.

Or

(b) Derive a t-distribution.

20. (a) Let X_i denote a random variable with mean μ_i and variance σ_i^2, $i = 1, 2, .. n$. Let $X_1, X_2, ..., X_n$ be independent and let $k_1, k_2, ... k_n$ denote real constants. Compute the mean and variance of $Y = k_1 X_1 + k_2 X_2 + ... + k_n X_n$.

Or

(b) State and prove the central limit theorem.
