(7 pages) **Reg. No. :** ....

Code No. : 7135 **Sub. Code : PMAM 43**

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2019.

Fourth Semester

Mathematics – IV

ADVANCED ALGEBRA – II

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. The number e is

   (a) rational (b) algebraic

   (c) trancendental (d) a unit

2. What is the degree of $\sqrt{2}\sqrt{3}$ over Q?

   (a) 1 (b) 2

   (c) 3 (d) 4

3. τ^* is an isomorphism of $F[x]$ onto $F'[t]$ with the property that, for all $\alpha \in F$, $\alpha\tau^* =$

   (a) α (b) 0

   (c) α' (d) t

4. If $f'(x)=0$ where $f(x)\in F[x]$ and f is of characteristic 3 then for some polynomial $g(x)\in F[x]$,

   (a) $g'(x)=0$ (b) $f(x^3)=g(x)$

   (c) $f(x)=g(x)$ (d) $f(x)=g(x^3)$

5. If $F(x_1, x_2, \ldots x_n)$ is the field of rational functions in $x_1, x_2 \ldots x_n$ over F and S is the field of symmetric rational functions then $[F(x_1, x_2 \ldots x_n):S]=$

   (a) S_n (b) n

   (c) $n!$ (d) $G(F(x_1, x_2, \ldots x_n), S)$

6. If F is the field of rational numbers and $K=F(\sqrt[3]{2})$ then $0(G(K,F))$ is

   (a) 1 (b) 2

   (c) 3 (d) 4

7. If F is a field with 9 elements, $F \subset K$ where K is a finite field such that $[K:F]=2$ then K has ———— elements.

(a) 7 (b) 18

(c) 512 (d) 81

8. The cyclotomic polynomial $P_6(x) =$

(a) x^2+x-1 (b) $x^4-x^3-x^2+1$

(c) x^2-x+1 (d) x^6-x^3+1

9. The irreducible polynomials over the field of real numbers are of degree

(a) 1 (b) 2

(c) either 1 or 2 (d) neither 1 nor 2

10. If $x \in H$, the Hurwitz ring of integral quaternions $x \neq 0$ then $N(x)$ is

(a) x^* (b) 0

(c) a positive integer (d) can't say

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If L is an algebraic extension of K and if K is an algebraic extension of F, show that L is an algebraic extension of F.

Or

(b) If $V=(g(x))$ is the ideal generated by the polynomial $g(x)$ of degree n in $F[x]$, prove that $\frac{F[x]}{V}$ is an n–dimensional vector space over F.

12. (a) Prove that a polynomial of degree n over a field can have at most n roots in any extension field.

Or

(b) Prove that the polynomial $f(x) \in F[x]$ has a multiple root if and only if $f(x)$ and $f'(x)$ have a nontrivial common factor.

13. (a) Define the fixed field of a group G of automorphsims of K and show that it is a sub field of K.

Or

(b) If K is a field and if $\sigma_1, \sigma_2, \ldots \sigma_n$ are distinct automorphisms of K, show that it is impossible to find elements $a_1, a_2 \ldots a_n$ not all O, in K such that $a_1\sigma_1(u)+a_2\sigma_2(u)+\ldots+a_n\sigma_n(u)=0$ for all $u \in K$.

14. (a) Show that for every prime number p and every positive integer m there exists a field having p^m elements.

Or

(b) If F is a finite field and $\alpha \neq 0$, $\beta \neq 0$ are two elements of F, show that there exist elements a and b in F such that $1 + \alpha a^2 + \beta b^2 = 0$.

15. (a) Show that the adjoint in the division ring Q of real quaternions satisfies the following :

(i) $x^{**} = x$

(ii) $(\delta x + \gamma y)^* = \delta x^* + \gamma y^*$

(iii) $(xy)^* = y^* x^*$ for all x, y in Q and all real δ and γ.

Or

(b) Define the norm $N(x)$ in Q and show that, for all x, y in Q $N(xy) = N(x)N(y)$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) If L is a finite extension of K and if K is a finite extension of F, show that $[L:F] = [L:K][K:F]$. Draw your inference when $[L:F]$ is a prime number.

Or

(b) If $a \in K$ is algebraic of degree n over F, prove that $[F(a):F] = n$.

17. (a) If $p(x)$ is irreducible in $F[x]$ and if V is a root of $p(x)$ then, show that $F(V)$ is isomorphic to $F'(W)$ where W is a root of $p'(t)$, by an isomorphism σ such that (i) $v\sigma = w$ and (ii) $\alpha\sigma = \alpha'$ for every α in F.

Or

(b) If F is of characteristics O and if a, b are algebraic over F, prove that there exists an element C in $F(a, b)$ such that $F(a, b) = F(c)$.

18. (a) Prove that $[K:F] = O(G(K, F))$, where K is a normal extension of F.

Or

(b) Given $F(x_1, x_2 \ldots x_n)$ is to field of rational functions, S is the field of symmetric rational functions $a_1, a_2, \ldots a_n$. Prove that (i) $S = F(a_1, a_2, \ldots a_n)$ and (ii) $F(x_1, x_2 \ldots x_n)$ is the splitting field over S of the polynomial $t^n - a_1 t^{n-1} + a_2 t^{n-2} + \ldots + (-1)^n a_n$.

19. (a) Given G is a finite abelian group with the property that $x^n = e$ is satisfied by at most n elements of G, for every integer n. Show that G is a cyclic group. Deduce that the multiplicative group of non zero elements of a finite field is cyclic.

Or

(b) State and prove Wedderburn's theorem on finite division kings.

20. (a) State and prove Frobenius theorem.

Or

(b) State and prove Lagrange's four-square theorem.
