(6 pages) **Reg. No. :** ....

Code No. : 7120 **Sub. Code : PMAM 24**

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2019.

Second Semester

Mathematics – Core

DIFFERENTIAL GEOMETRY

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer:

1. $[r', r'', r'''] =$ ——————.

(a) k^2 (b) $k^2\tau$

(c) $k\tau$ (d) $k^3\tau$

2. The line of intersection of the normal plane and the osculating plane at P is the

(a) Normal plane (b) Tangent plane

(c) Principal plane (d) Principal normal

3. The pitch of the helix is ——————.

(a) $2\pi a$ (b) $2\pi b$

(c) ab (d) πb

4. If a curve lies on a sphere, then $\frac{d}{ds}(\sigma p') + \frac{p}{\sigma} =$ ——————.

(a) 1 (b) σ

(c) p (d) 0

5. —————— point is defined as one for which $r_1 \times r_2 \neq 0$.

(a) Ordinary (b) Singular

(c) Osculating (d) Non-ordinary

6. The normal component of a is given by $a_n =$ ——————.

(a) $a \cdot b$ (b) $a \times b$

(c) $a \cdot N$ (d) $a \times N$

7. The two families are orthogonal if an only if ——————.

(a) $ER - FQ + GP = 0$

(b) $ER - 2FQ + GP = 0$

(c) $EP - 2FQ + GR = 0$

(d) $ER + 2FQ + GP = 0$

8. A necessary and sufficient condition for a Geodesic is ______.

(a) $U\frac{\partial T}{\partial \dot{v}} - V\frac{\partial T}{\partial \dot{u}} = 0$ (b) $U\frac{\partial T}{\partial \dot{v}} - V\frac{\partial T}{\partial \dot{u}} \neq 0$

(c) $U\frac{\partial T}{\partial \dot{v}} + V\frac{\partial T}{\partial \dot{u}} = 0$ (d) $U\frac{\partial T}{\partial \dot{v}} + V\frac{\partial T}{\partial \dot{u}} \neq 0$

9. The geodesic curvature of a Geodesic is ______.

(a) 1 (b) 0

(c) H (d) E

10. A point where $\frac{L}{E} = \frac{M}{F} = \frac{N}{G}$ is called ______.

(a) An ordinary (b) An essential

(c) An unbilic (d) A singularity

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Calculate the curvature and torsion of the cubic curve given by $r = (u, u^2, u_3)$.

Or

(b) State and prove Serret – Frenet formulae.

12. (a) Explain the oscillating circle.

Or

(b) Explain the oscillating sphere.

13. (a) For the paraboloid, $x = u$, $y = v$, $z = u^2 - v^2$, compute the values of E, F, G and H.

Or

(b) Discuss about the general helicoids.

14. (a) On the paraboloid $x^2 - y^2 = z$, find the orthogonal trajectories of the sections by the planes $z =$ constant.

Or

(b) A helicoids generated by the screw motion of a straight line which meets the axis at an angle α. Find the orthogonal trajectories of the generators.

15. (a) Prove that if the orthogonal trajectories of the curve $v =$ constant are geodesics, then H^2 / E is independent of u.

Or

(b) Derive the formula

$$K_g = \frac{1}{H_{S'}}\left(\frac{\partial T}{\partial \dot{u}} V(t) - \frac{\partial T}{\partial \dot{v}} U(t)\right).$$

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) For the two quadratic surfaces, $ax^2 + by^2 + cz^2 = 1$ and $a'x^2 + b'y^2 + c'z^2 = 1$, Obtain the curvature and torsion of the curve of intersection.

Or

(b) Define (i) a curve at class in E_3. (ii) Binormal line at P. (iii) Change of parameter. (iv) Principal normal.

17. (a) Discuss the cylindrical helix.

Or

(b) Discuss about the circular helix.

18. (a) Find the coefficient of the direction which makes an angle $\pi/2$ with the direction whose coefficients are (l, m).

Or

(b) Define (i) anchor ring (ii) representation (iii) Circular helix. (iv) Singularity.

19. (a) Prove that on the curves of the family $v^3/u^2 =$ constant are geodesics on a surface with metric $v^2du^2 = 2uvdudv + 2u^2du^2$, $(u > 0, u > 0)$.

Or

(b) Prove that on the general surface a necessary and sufficient condition that the curve $v = c$ be a geodesic is $EE_2 + FE_1 - 2EF_1 = 0$.

20. (a) State and prove Liouvell's formula for K_g.

Or

(b) Derive the Rodrigue's formula for the lines of curvature.
