(7 pages) **Reg. No. : ....**

Code No. : 7117 **Sub. Code : PMAM 21**

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2019.

Second Semester

Mathematics – Core

ALGEBRA – II

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

SECTION A — (10 × 1 = 10 marks)

Answer ALL the questions.

Choose the correct answer :

1. If F is a field, the number of ideals of F is

   (a) 1 (b) 2

   (c) at least 2 (d) 0

2. The homomorphism ϕ of R into R' is an isomorphsim if and only if

   (a) $I(\phi) \neq (0)$ (b) $I(\phi)$ is an ideal of R

   (c) $I(\phi) = (0)$ (d) $I(\phi) = R$

3. If π is a prime element in the Euclidean ring R and $a \in R$ then

   (a) $(\pi, a) = 1$

   (b) a / π

   (c) If $\pi \nmid a$ then $(\pi, a) = 1$

   (d) If π / a then $(\pi, a) = 1$

4. A solution of $x^2 \equiv -1(\text{mod}13)$ is

   (a) 6 (b) 5

   (c) 3 (d) 2

5. The degree of $5 + 7x^2 + 4x^4 + 11x^5$ over the integers mod 11 is

   (a) 4 (b) 5

   (c) 0 (d) 2

6. Which one of the following is not true?

   (a) A Euclidean ring is a unique factorization domain

   (b) A Euclidean ring is a principal ideal ring

   (c) If F is a field, $F[x_1, x_2]$ is a principal ideal ring

   (d) If R is an integral domain then so is $R[x]$

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7. If $R = \mathbf{Z}$, the ring of integers then rad R is

(a) $\mathbb{Z}$

(b) (0)

(c) (P) for same prime number P

(d) $2\mathbf{Z}$

8. The relation between rad R and Rad R is

(a) $Rad\ R = rad\ R$

(b) $Rad\ R \subseteq rad\ R$

(c) $rad\ R \subseteq Rad\ R$

(d) They are not comparable

9. A ring R is isomorphic to a sub direct sum of integral domains if and only if

(a) R is semi simple

(b) R is without prime radical

(c) R is a ring without identify

(d) R is a commutative ring

10. For any commutative regular ring R, $J(R)$ is

(a) ϕ (b) $\{0\}$

(c) R (d) the centre of R

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If U, V are ideals of R, let $U + V = \{u + v / u \in U, v \in V\}$. Prove that $U + V$ is also an ideal.

Or

(b) Let R be a commutative ring with unit element whose only ideals are (O) and R itself. Prove that R is a field.

12. (a) Let R be a Euclidean ring. Prove that any two elements a and b in R have a greatest common division and $d = \lambda a + \mu b$ for same $\lambda, \mu \in R$.

Or

(b) Prove that $J[i]$ is a Euclidean ring.

13. (a) If $f(x), g(x)$ are two new zero elements of $f[x]$, prove that $\deg(f(x)\ g(x)) = \deg f(x) + \deg g(x)$.

Or

(b) State and prove Gauss's lemma.

[P.T.O.]

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14. (a) Let R be a principal ideal domain. Prove that R is semi simple if and only if R is either a field on has an infinite number of maximal ideals.

Or

(b) For any ring R, prove that the quotient ring $R/Rad\ R$ is without prime radical.

15. (a) Prove that an element a of the ring R is quasi regular if and only if there exists some $b \in R$ such that $a+b-ab=0$.

Or

(b) Let R be a ring containing no non zero nil ideals. Prove that R is isomorphic to a sub direct sum of integral domain.

SECTION C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) If U is an ideal of the ring R, prove that R/U is a ring and is a homomorsphic image or R.

Or

(b) Prove that every integral domain can be imbedded in a field.

17. (a) Prove that the ideal $A=(a_0)$ is a maximal ideal of the Euclidean ring R if and only if a_0 is a prime element or R.

Or

(b) After proving the necessary lemmas, prove that if p is a prime number of the form $4n+1$, then $p=a^2+b^2$ for some integer a,b.

18. (a) State and prove the Eisenstein criterion.

Or

(b) If R is a unique factorization domain, prove that $R[x]$ is also a unique factorization domain.

19. (a) If I is an ideal of the ring R, prove that

(i) $rad(R/I) \supseteq \frac{rad\ R+I}{I}$ and

(ii) whenever $I \subseteq rad\ R$, $rad\left(\frac{R}{I}\right)=$ $(rad\ R)/I$.

Or

(b) Define a primary ring. Prove that a ring R is a primary ring if and only if R has a minimal prime ideal which contains all zero divisions.

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20. (a) Prove that a ring R is isomorphic to a sub direct sum of ring Ri, if and only if R contains a collection of ideals $\{Ii\}$ such that $R/I_i \simeq R_i$ and $\bigcap I_i = (0)$.

Or

(b) Prove that every ring R is isomorphic to a sub direct of sum of sub directly irreducible rings.

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