(7 pages) **Reg. No. : ....**

Code No. : 7120 **Sub. Code : PMAM 22**

M.Sc. (CBCS). DEGREE EXAMINATION, APRIL 2018.

Second Semester

Mathematics

ANALYSIS – II

(For those who joined in July 2017 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. Let $f \in \mathcal{R}_a(\alpha)$ be a function defined on $[a,b]$ then
   (a) $|f| \in \mathcal{R}_a(\alpha)$ (b) $-f \in \mathcal{R}_a(\alpha)$
   (c) $cf \in \mathcal{R}_a(\alpha)$ (d) All are true

2. The value of the unit step function I defined on the set of positive integers is
   (a) 0 (b) 1
   (c) Positive value (d) None

3. The curve γ is rectifiable if
   (a) $\Lambda(\gamma) = \infty$ (b) $\Lambda(\gamma) = 0$
   (c) $\Lambda(\gamma) < \infty$ (d) None

4. For $m = 1,2,3..$ $n = 1,2,3...$ then $\lim_{m\to\infty}\lim_{n\to\infty} \frac{m}{m+n}$ is
   (a) 1 (b) 0
   (c) Does not exist (d) None

5. Let $f_n(x) = \frac{x^2}{x^2 + (1-nx)^2}$ $0 \le x \le 1, n = 1,2,3...$

   Then $\{f_n\}$ is
   (a) equi continuous on [0,1]
   (b) uniformly bounded on [0,1]
   (c) unbounded
   (d) none

6. Which of the following is not true
   (a) Every member of an equi continuous family is uniformly continuous
   (b) Every pointwise bounded sequence is uniformly bounded
   (c) Every uniformly bounded sequence is point wise bounded
   (d) None

7. $\lim_{n\to\infty}\left(1+\frac{x}{n}\right)^n$ is

(a) e^{-x} (b) e^x

(c) e^{-1} (d) None

8. $\lim_{x\to 0}\frac{\tan x - x}{x(1-\cos x)}$ is

(a) 0 (b) 1

(c) ∞ (d) None

9. An orthogonal system $\{\phi_n\}$ is orthononmal if

$\int_a^b |\phi_n(x)|^2\, dx$ is

(a) 0 (b) 1

(c) $b-a$ (d) a

10. The value of $\int_{-\infty}^{\infty} e^{-x^2 dx}$ is

(a) $\frac{\pi}{2}$ (b) $\frac{\sqrt{\pi}}{2}$

(c) $\sqrt{\pi}$ (d) None

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If $f_1 \in \mathcal{R}(\alpha)$ and $f_2 \in \mathcal{R}(\alpha)$ on $[a b]$ then prove that $f_1 + f_2 \in \mathcal{R}(\alpha)$ on $[a b]$ and

$$\int_a^b f_1 + f_2\, d\alpha = \int_a^b f_1\, d\alpha + \int_a^b f_2\, d\alpha$$

Or

(b) State and prove fundamental theorem of calculus.

12. (a) For any metrix space X, prove that $\mathscr{C}(X)$, space of all complex valued bounded functions defined on X is a complete metric space.

Or.

(b) Give an example to show that the limit of the integral need not be equal to the integral of the limit.

13. (a) If $\{f_n\}$ is a uniformly bounded sequence of continuous functions on a compact set E, then prove that there need not exist a subsequence which converges point wise on E.

Or

(b) If K is a compact metric space if $f_n \in \mathscr{C}(K)$ for $n = 1,2...$ and if $\{f_n\}$ converges uniformly on K then prove that $\{f_n\}$ is equi continuous on K.

14. (a) Given a double sequence $\{aij\}$ $i = 1,2,3...$ $j = 1,2,3...$ Suppose that $\sum_{j=1}^{\infty} |aij| = bi (i = 12...)$ and Σbi converges then prove that $\sum_{i=1}^{\infty}\sum_{j=1}^{\infty} aij = \sum_{j=1}^{\infty}\sum_{i=1}^{\infty} aij$

Or

(b) State and prove Taylor's theorem.

15. (a) If f is continuous with period 2π and if $\in > 0$ then show that there is a trigonometric polynomial P such that $|P(x) - f(x)| < \in$ for all x.

Or

(b) If $x > 0, y > 0$ then prove that

$$\int_0^1 t^{x-1}(1-t)^{y-1}\,dt = \frac{\Gamma x \Gamma y}{\Gamma x + y}.$$

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Suppose $f \in \mathscr{R}(\alpha)$ on $[a,b] m \le f \le m, \phi$ is continuous on $[m, M]$ and $h(x) = \phi(f(x))$ on $[a,b]$. Then prove that $h \in \mathscr{R}(\alpha)$ on $[a,b]$

Or

(b) Assume α increases monotonically and $\alpha' \in \mathscr{R}$ on $[a,b]$. Let f be a bounded real function on $[a,b]$. Then prove that $f \in \mathscr{R}(\alpha)$ if and only if $f\alpha' \in \mathscr{R}$. In that case $\int_a^b f d\alpha = \int_a^b f(x)\alpha'(x)\,dx$.

17. (a) If γ' is continuous on $[a,b]$ then prove that γ is rectifiable and $\Lambda(\gamma) = \int_a^b |\gamma'(t)|\,dt$.

Or

(b) If $\{f_n\}$ is a sequence of continuous function on E and if $f_n \to f$ uniformly on E. Then prove that f is continuous on E.

18. (a) If K is compact if $f_n \in \mathscr{C}(K)$ for $n = 1,2,3..$ and if $\{f_n\}$ is point wise bounded and equicontinuous on K then prove that

(i) $\{f_n\}$ is uniformly bounded on K .

(ii) $\{f_n\}$ contains a uniformly convergent subsequence.

Or

(b) Prove that there exists a real continuous function on the real line which is nowhere differentiable.

19. (a) State and prove stone's generalization of the Weierstrass theorem.

Or

(b) State and prove stone Weierstrass theorem.

20. (a) State and prove Parseval's theorem.

Or

(b) State and prove stirlings formula.

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