Functions

Defn

If $f: A \to B$, $|A| = n$, $|B| = m$.

$\therefore$ Total no. of maps $= m^n = |B|^{|A|}$

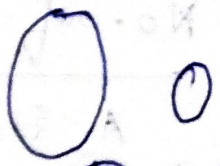
If $n < m$

1-1 map $= {}^mC_n \cdot n! = \frac{m!}{(m-n)!}$ (another way backside page) $\frac{m!}{(m-n)!\,n!}$

onto → not possible

Bijection → not possible.

If $n > m$



1-1 map → not possible.

Bijection → not possible

onto → $m^n - \{{}^mC_1 (m-1)^n - {}^mC_2 (m-2)^n + {}^mC_3 (m-3)^n - \dots\}$

If $n = m$

1-1 map → $n!$

onto → $n!$

Bijection → $n!$

$|A| = n \quad |B| = m$

If $n \le m$

Total 1-1 maps

$= m(m-1)(m-2)\cdots(m-(n-1))$

$= m(m-1)(m-2)\cdots(m-n+1)$

ex $|A| = 5 \quad |B| = 8$

No. of 1-1 maps

$= 8(7)(6)\cdots(8-5+1)$

$= 8 \times 7 \times 6 \times 5 \times 4$

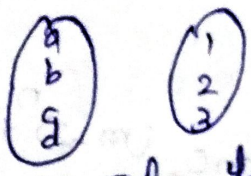
$=$

(or)

No. of 1-1 maps

$$\frac{8!}{(8-5)!} = \frac{1.2.3.4\cdots8}{1.2.3} = (4 \times 5 \times 6 \times 7 \times 8)$$

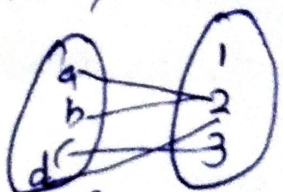
$n \ge m$ No. of onto funs



$\therefore$ T no. of funs is 3^4.

and We remove from the total no. of funs. which are not onto,

Consider,



← left 1 from one element

this is not onto this case

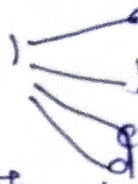
The total no. of funs is,

No. of 1-1 funs

$A = \{1, 2, 3\}$

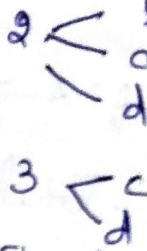
$B = \{a, b, c, d\}$

The choices of 1 is,



suppose $1 \to a$

then The choices of 2 is,



3 choices if $2 \to b$

2 choices

$\therefore$ total possibilities of 1-1 is

$4.3.2 = 24$.

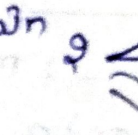
total No. of funs

$A = \{1, 2, 3\} \quad B = \{a, b, c, d\}$

The choices of 1 is,

4 choices $1 \to a$ (if)

The choices of 2 is again



4 choices

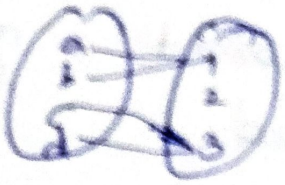
2 can map a also

Also 3 has 4 choices

$\therefore$ total choices are 4^3.

case (ii)

Remove 2 (left 1 one element)



again 2^3 funs

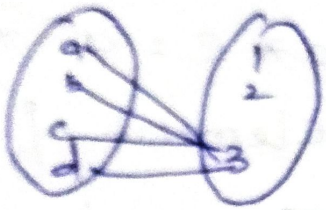
case (iii)

III^{rly} for (3)

again we have

2^4 funs

If we remove 2 elements, then



suppose remove 1, & 2

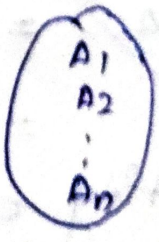
$\therefore$ 1^4 funs.

ie 1 fun

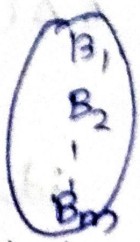
$\therefore$ Total no. of ~~no~~ onto

$$= 3^4 - \{ {}^3C_1 (2^4) + {}^3C_2 (1)^4 \}$$

$$n(B_1 \cup B_2 \cup \dots \cup B_m) = \sum n(B_1) - \sum n(B_1 \cap B_2) + n(B_1 \cap B_2 \cap B_3) - \dots$$



$|A| = n$



$|B| = m$. $(n \times m)$